From experience to math

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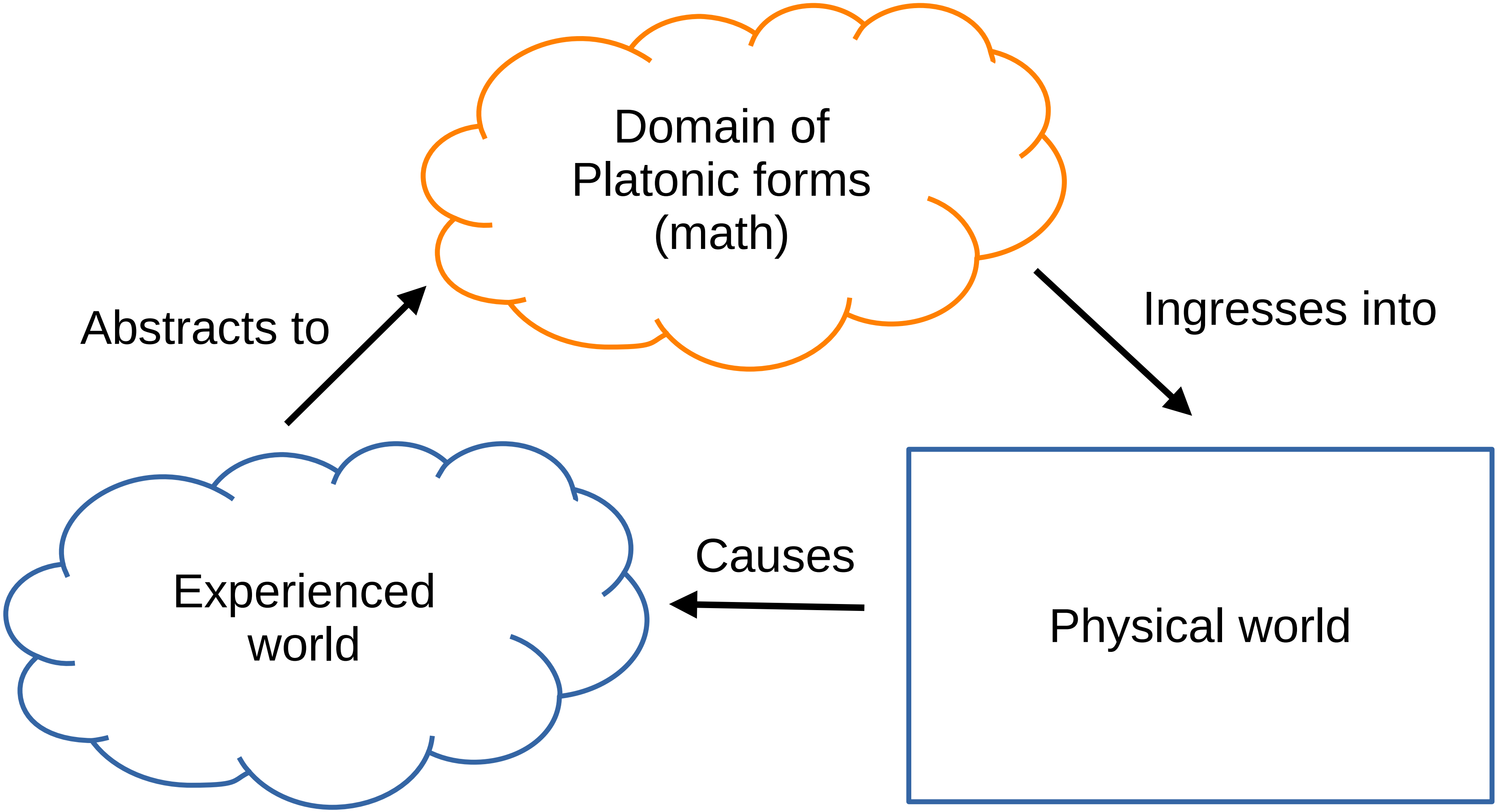
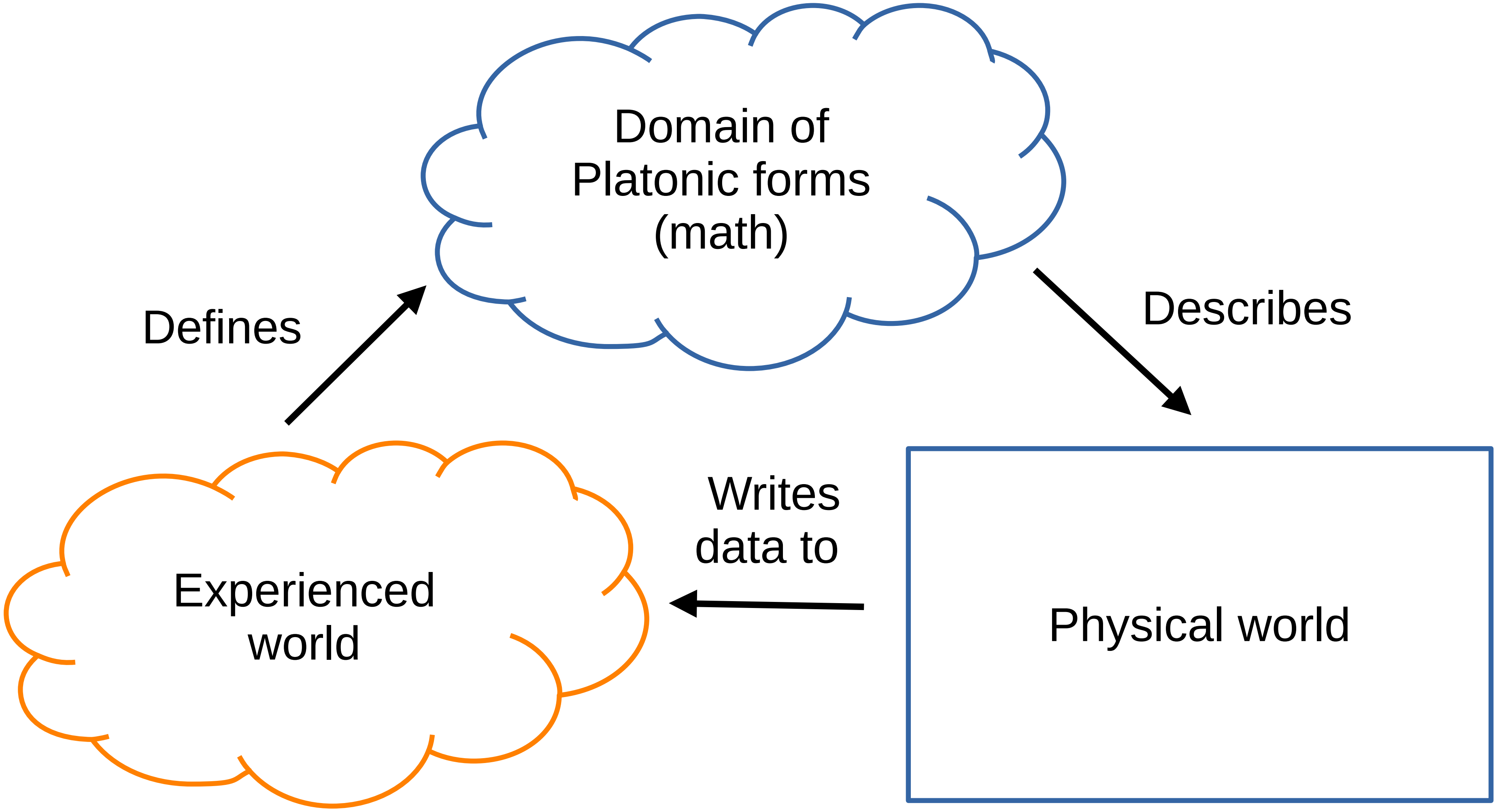

Domain of
Platonic forms
(math)
Abstracts to
Ingresses into
Experienced
world
Causes
Physical world


Domain of
Platonic forms
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Defines
Describes
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Physical world

Who has the same math that we do?

Platonic: Everybody, by definition. We can only find out how much they know if we share communication capabilities.

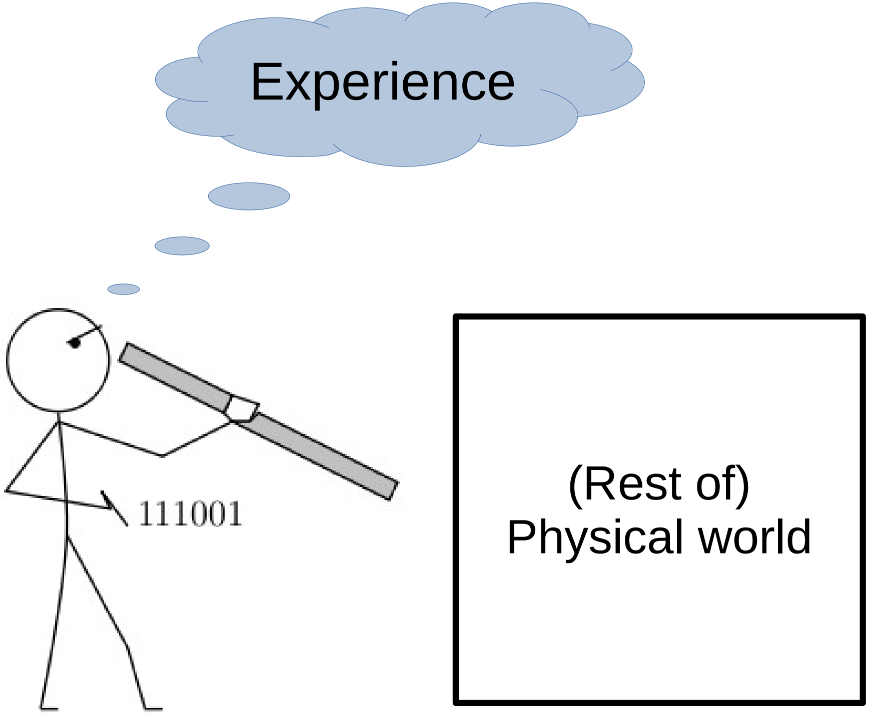
Experiential: Systems with which we share experiential capabilities. Communication follows from shared experience.

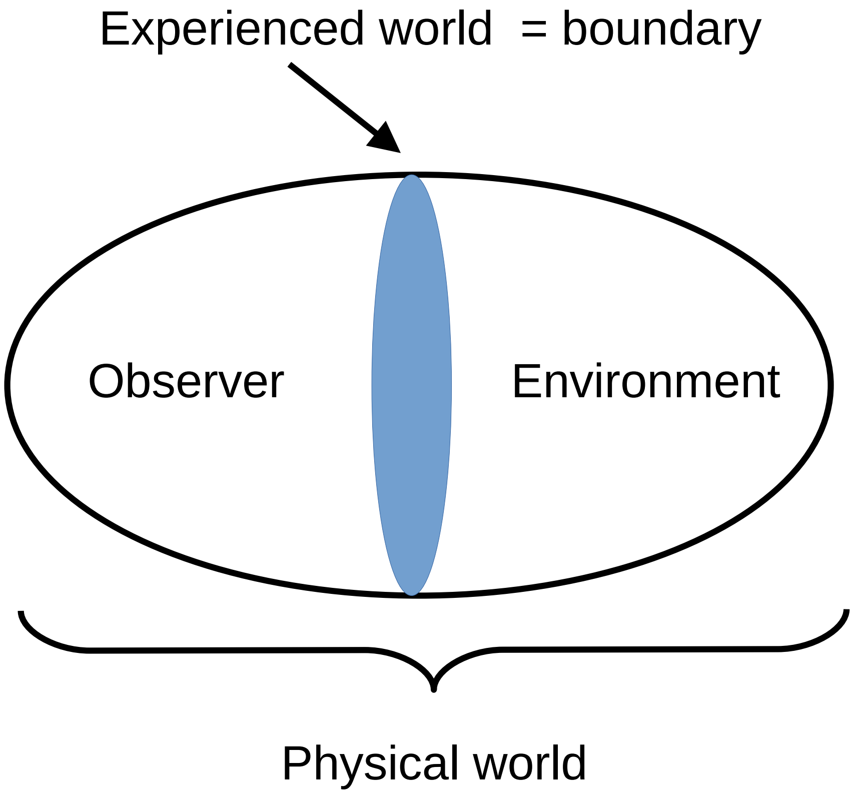
We’re changing the perspective, not the facts.

Where we're going …

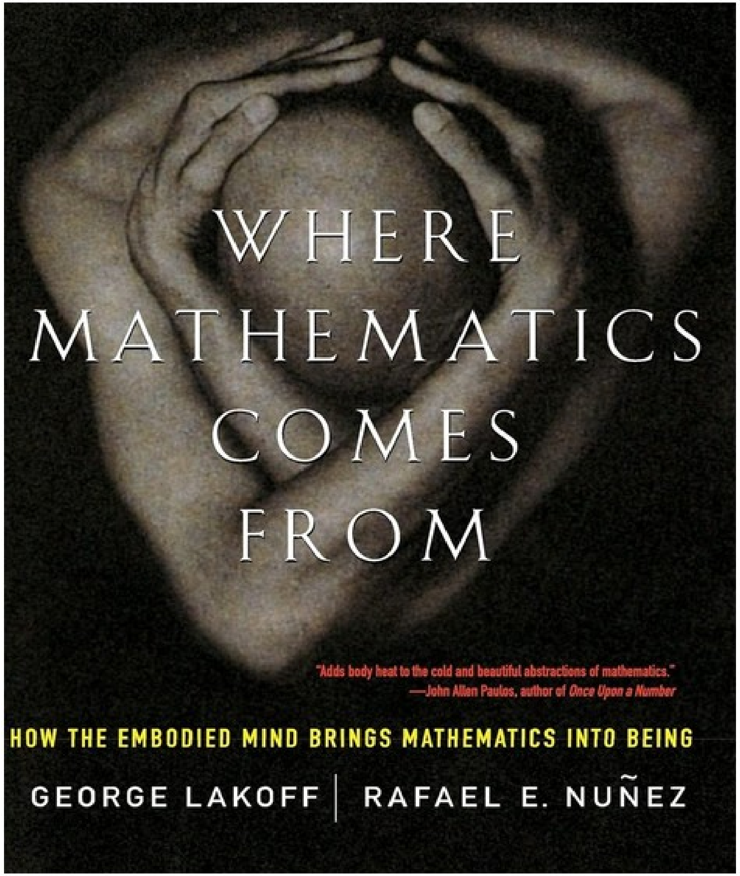
- What is the experienced world?
- Experience ⟶ categories
- Categories ⟶ math as we know it
  - Example: prime numbers
  - Example: qubit spaces
  - Example: computation
- Proofs
- Why is this so hard?
- What do we learn from this perspective?

Two “pictures” of experience





What kinds of experiences are needed for math?



Claim:

The experience of *identity over time* is the key to mathematics.

“Don’t take it for granted, abstract it!”

Let’s start by building some basic math

Experiences	Axioms
The world contains objects.	There is a class **O** of “objects”.
If nothing happens to them, objects persist through time.	For each object A in **O**, there is an “identity” map id_A: A → A.

Note that `→’ indicates the passage of “time”.

Now for processes ...

Experiences	Axioms
Objects can undergo processes of change	For A, B in **O**, there is a set (maybe Ø) of "morphisms" f: A → B
All but the simplest processes of change have intermediate states	If f: A → B and g: B → C, there is a map gf: A → C
Only the order of the intermediates matters	If f: A → B and g: B → C and h: C → D, $h(gf) = (hg)f$

A *category* $\mathfrak{C}$ is a quadruple $(\mathcal{O}, \hom, id, \circ)$ consisting of:

1. a class $\mathcal{O}$ of *objects* (or $\mathfrak{C} - objects$),
2. for each pair (A, B) of objects, a set $\hom(A, B)$ of *morphisms* $f : A \to B$,
3. for each object A, a morphism $id_A : A \to A$,
4. a composition $\circ$ such that if $f : A \to B$ and $g : B \to C$ there is a morphism $f \circ g : A \to C$, also written gf.

subject to the following conditions:

a) composition is associative, i.e. if $f : A \to B$, $g : B \to C$, and $h : C \to D$, then $h(gf) = (hg)f$.

b) composition respects identities, i.e. if $f : A \to B$, $id_B f = f$ and $f id_A = f$, and

c) the sets $\hom(A, B)$ are pairwise disjoint.

Adámek, J., Herrlich, H. and Strecker, G. E. *Abstract and Concrete Categories: The Joy of Cats*, 2004, p. 21.

Some other correspondences ...

Life experiences	Category theory
There is usually more than one way to get something done	All interesting categories have commutative diagrams
Sometimes actions can be reversed, and when they can be, things are easier	Some morphisms have adjoints, and these make things easier
The order in which things are done often matters	Morphisms that commute, e.g. $fg = gf$, are special cases

“Everything else is a special case ...”

Sets, groups, rings, fields, vector spaces, topological spaces, etc are specifications of properties of objects and morphisms.

They all exemplify the basic intuitions that define categories:

Objects have identities
Processes compose
Order matters

What they add are “elements” in objects and various “operations” on these elements.

Category theory $\longrightarrow$ Math as we know it

Key intuition: Composition is the fundamental binary operation.

Suppose we have:

$$X \xrightarrow{f} X \xrightarrow{f} X \xrightarrow{f} X \xrightarrow{f \ldots} \cdots$$

so we have, e.g. *f*, *ff, fff, ffff*, …

This looks like concatenation, or addition.

So we could think of “1” as “do *f* once” etc. Numbers are *actions*.

Example: prime numbers

Doing 1:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	…
Doing 2:		2		4		6		8		10		12		14		16		18	…
Doing 3:			3						9						15				...
Doing 4:									nothing new										
Doing 5:					5									nothing new till 25					
Doing 6:									nothing new										
Doing 7:							7							nothing new till 49					
	1	2	3		5		7				11		13				17		

Prime numbers just need concatenation, i.e. composition.

So they are implicit in morphism composition and associativity –
the idea of putting parenthesis (boundaries) around things.

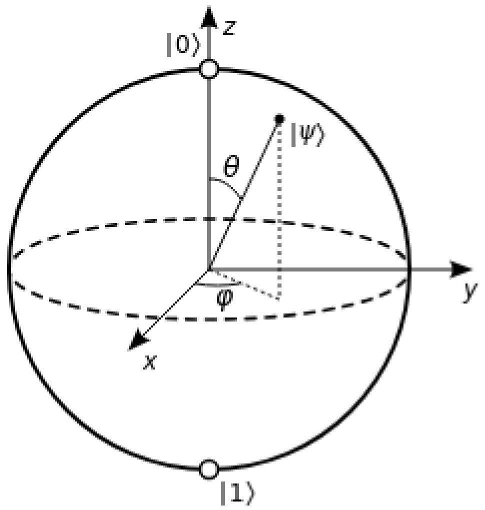
Example: qubit spaces

A qubit (quantum bit) is a physical system that, when measured with respect to a reference direction z, can be in one of two states, conventionally called |↑> or |↓>, or |+1> or |-1>.

An n-qubit space is a normalized real vector space where the vectors are n-tuples ($\varphi_1 \ldots \varphi_n$), $0 \le \varphi_i \le 2\pi$. We are typically interested in qubits parameterized by "time" t, so the vectors are ($\exp(-(i/\hbar)\varphi_1 t) \ldots \exp(-(i/\hbar)\varphi_n t)$). These are "quantum states".

Quantum theory is the theory of t-symmetric (unitary) linear maps from a qubit space to itself. These maps represent time evolutions.

Math can be misleading



Qubits are often represented by *Bloch spheres* in (x,y,z) Euclidean 3-space.

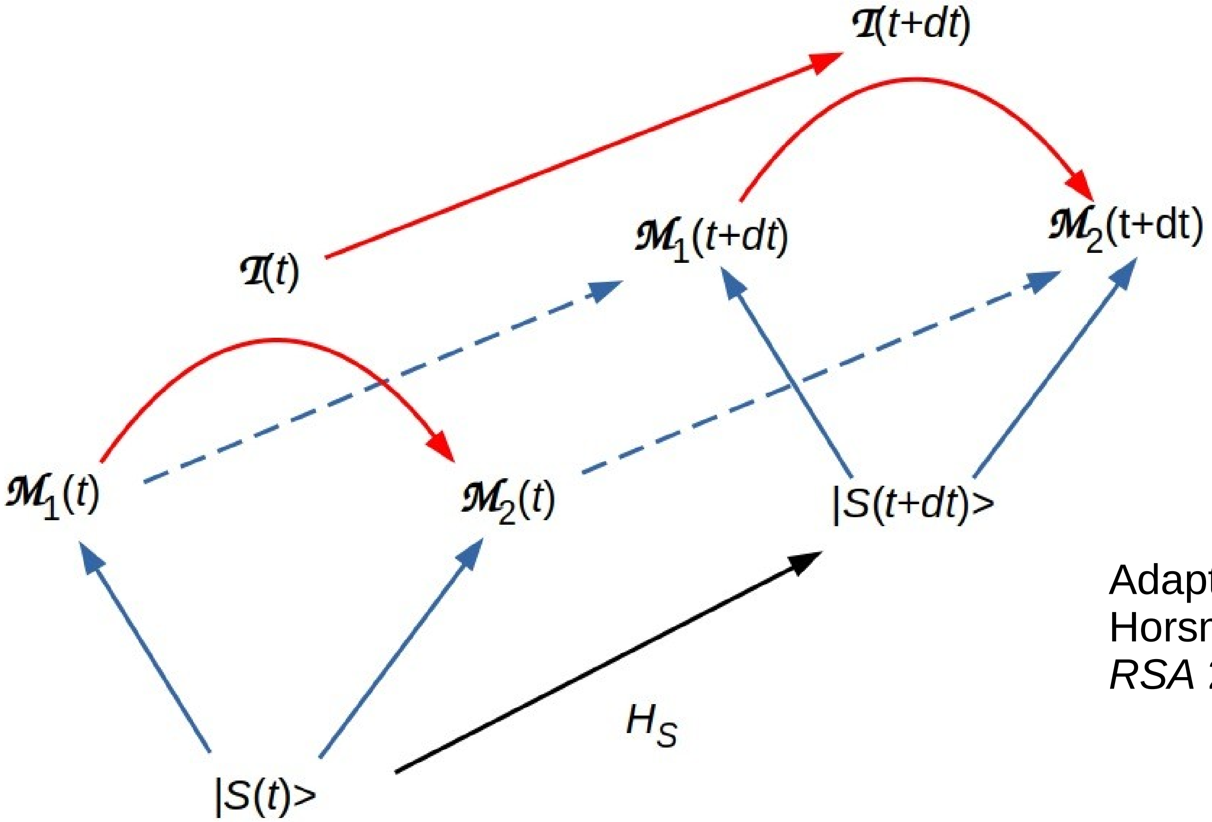
Where did these spatial coordinates come from?

Why two angles, θ and φ, instead of just one?

The x, y, and z directions in the Bloch sphere are the three orthogonal ways that we, in our labs, can choose the reference direction “z” in the definition of a qubit.

The Bloch sphere represents *our measurement capabilities* (reference frames) in the special case of 3d spatial orientation.

Example: Computing



Adapted from
Horsman et al.
RSA 2014

A *physically-implemented computation* is a commutative diagram.

Math can make assumptions explicit

When we make measurement and interpretation explicit, we see:

1. Interpreting something as computing requires computing a function.

   - We haven’t defined “computation”.
   - We can’t interpret a process as “computing” something non-computable by us.

2. Many interpretations are possible ⟶ polycomputing, VMs.

3. Computations is relative to environmental constraints – the environment shapes the free-energy landscape.

Proofs (What is logic?)

$(A, (A \rightarrow B)) \rightarrow B$

Entities have “causal power” as well as identity.
This causal power is context-independent.

$\neg(\neg A) \rightarrow A$

We are effectively assuming that everything is decidable.

$\exists$, $\exists!$, $\forall = \neg\exists\neg$

Membership in classes is decidable.

Context-independence and decidability are big assumptions!

Where did these assumptions come from?

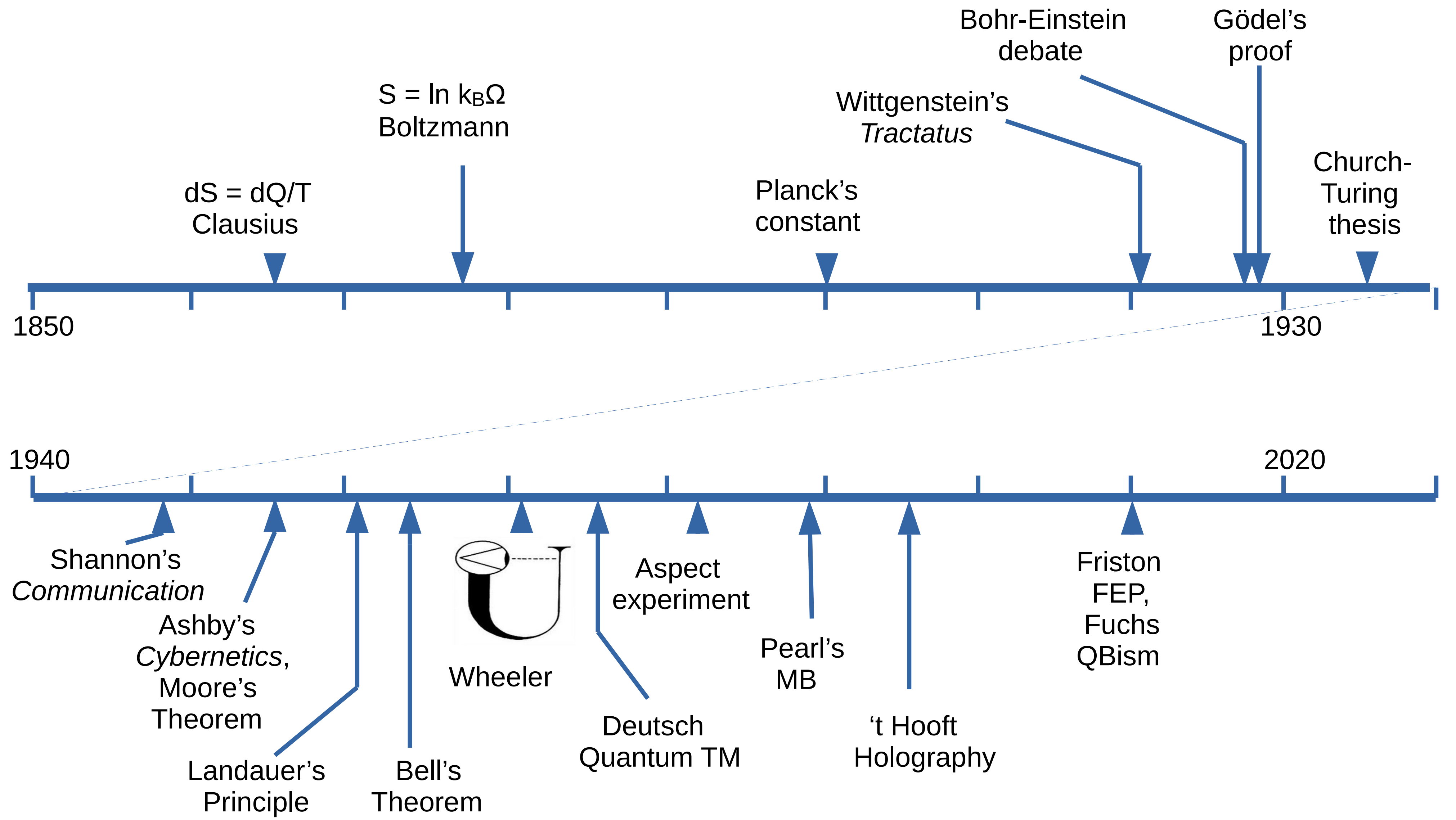
Newton limited non-locality to gravity.

Einstein forbade non-locality altogether.

Post-Einstein, classical physics assumed local, context-free events observed by effectively omniscient (real numbers), effectively omnipotent (isolated systems) observers.

1st-order logic and Hilbert's program embody these assumptions.

Gödel's theorem ("truth" involves hidden assumptions), non-computability (set membership often isn't decidable), and the Frame Problem (context matters) now look inevitable!


Bohr-Einstein debate
Gödel's proof
$S = \ln k_B\Omega$ Boltzmann
Wittgenstein's *Tractatus*
Church-Turing thesis
dS = dQ/T Clausius
Planck's constant
1850
1930
1940
2020
Shannon's *Communication*
Ashby's *Cybernetics*, Moore's Theorem
Wheeler
Aspect experiment
Friston FEP, Fuchs QBism
Pearl's MB
Landauer's Principle
Bell's Theorem
Deutsch Quantum TM
't Hooft Holography

Why is math hard? Why is it nonetheless natural?

Classical physics notwithstanding, the world is not transparent.

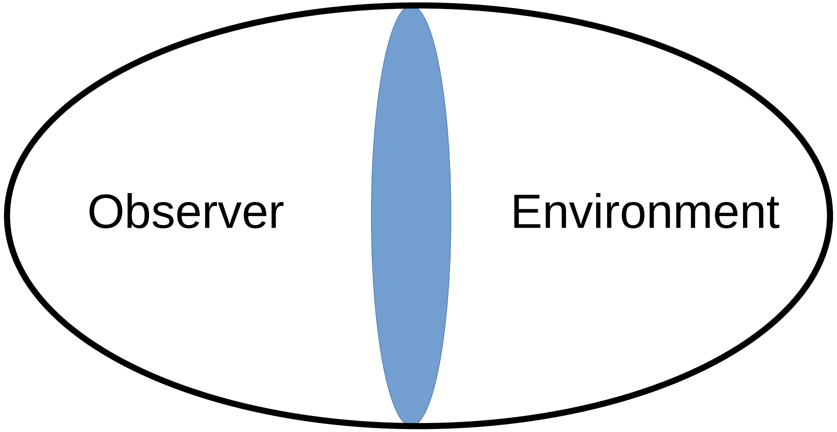
GOFAI notwithstanding, 1st-order logic from fixed axioms is not a good description of how our minds work.

But math is mainly about analogies, and we are good analogy machines.

Analogy is what our motor-planning systems do. Analogy is *how* we can achieve the same goal by multiple paths.

Example: History dependence (geometric phase)

We experience the actions of a high-dimensional environment on a low-dimensional boundary.



Paths in the environment map many-to-one to paths on the boundary. We only observe paths on the boundary.

Computers are useful precisely because they have lots of internal states! We don't have to watch everything they do.

Example: Functors

If $\mathfrak{A}$ and $\mathfrak{B}$ are categories, a *functor* $\mathcal{F} : \mathfrak{A} \to \mathfrak{B}$ is a function that assigns to each $\mathfrak{A}$-object A a $\mathfrak{B}$-object $\mathcal{F}(A)$, and to each $\mathfrak{A}$-morphism $f : A \to A'$ a $\mathfrak{B}$-morphism $\mathcal{F}(f) : \mathcal{F}(A) \to \mathcal{F}(A')$ such that:

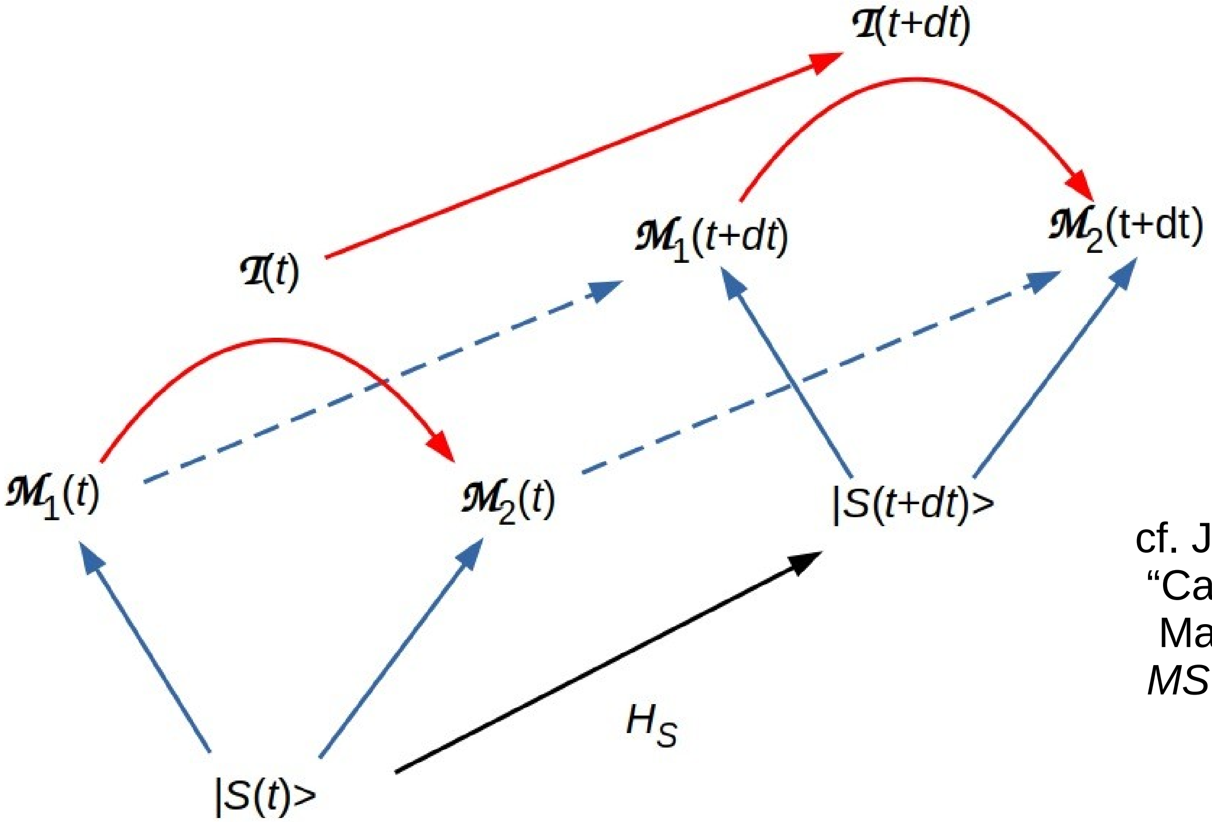
a) $\mathcal{F}$ preserves composition, i.e. $\mathcal{F}(gf) = \mathcal{F}(g)\mathcal{F}(f)$ whenever gf is defined.

b) $\mathcal{F}$ preserves identity morphisms, i.e. $\mathcal{F}(id_A) = id_{\mathcal{F}(A)}$ for each $\mathfrak{A}$-object A.

Joy of Cats, p. 29-30

Functors are structure mappings – D. Gentner’s term – i.e. well-formed analogies between mathematical systems. Like analogies in general, they can preserve more or less structural information when connecting two domains. Are all analogies functors? Are all domains formalizable?

Semantic maps are functors



cf. J. Goguen's "Categorical Manifesto" *MSCS*, 1991

A well-defined semantics is a useful analogy.

What are we doing when we’re doing math?

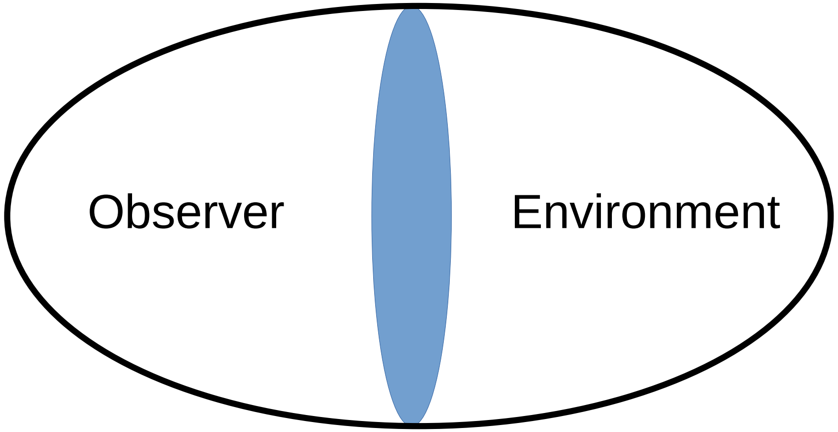
Constructing well-defined analogies.

Exploring the space of possible semantics/interpretations

“Playing with language” to find a useful description

All in the service of understanding/modeling observations
of processes unfolding in time, i.e. of active inference.

The “unreasonable effectiveness of mathematics”



Our environment is basically the same kind of entity that we are.

Our processes are analogs of its processes, because both are instances of active inference acting on the same boundary data.

Who has the same math that we do?

Our environment has the same kind of math!

Any system we can observe has the same kind of math.

Mutual observation/interaction *is* communication.

The “Platonic realm” is the world, including us.

Thank you

Questions?