National Institute of Environmental Health Sciences

Mathematics, morphogenesis, and metaphysics: thinking about Platonic space in biology

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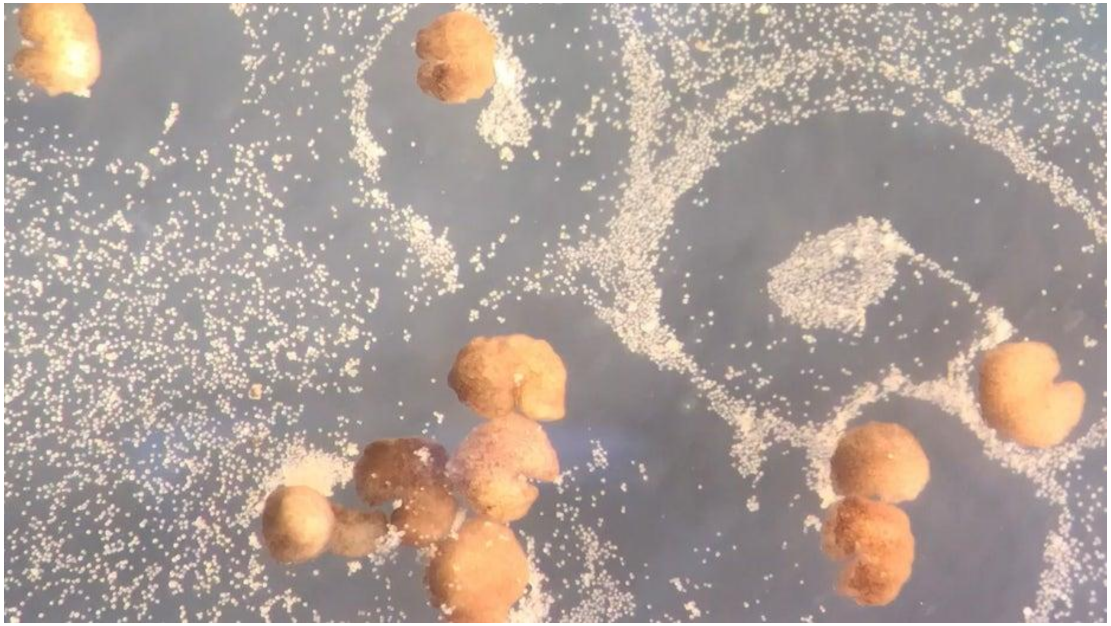
What motivates talk of “Platonic space: xenobots

https://www.youtube.com/shorts/8zH4LGllrnU

Novel life forms initiating morphologies and behaviors that have (as far as we know) no basis in evolution.

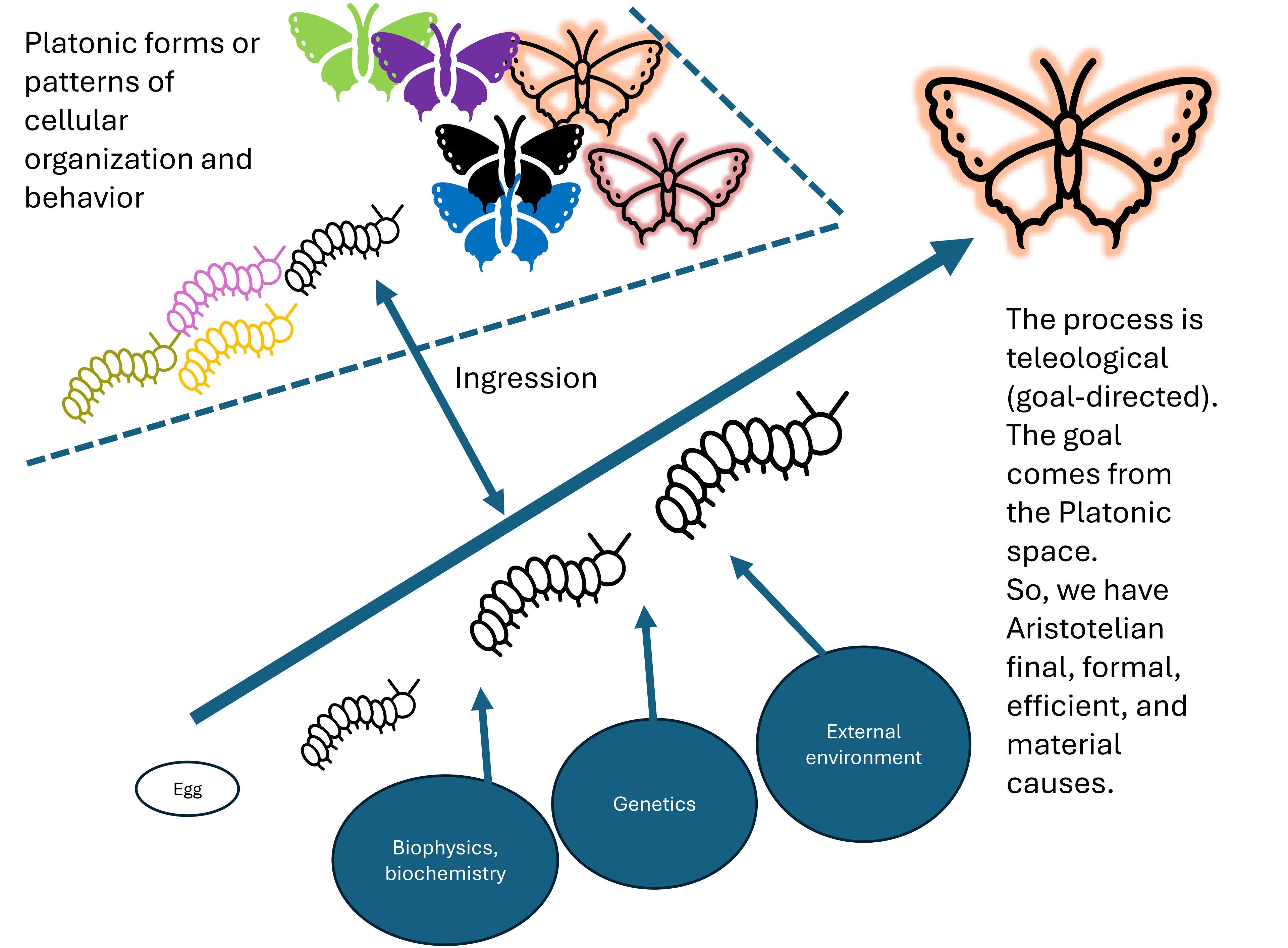
Where did the information come from to do this?



https://www.fox10phoenix.com/news/xenobots-worlds-first-living-robots-can-reproduce-scientists-say

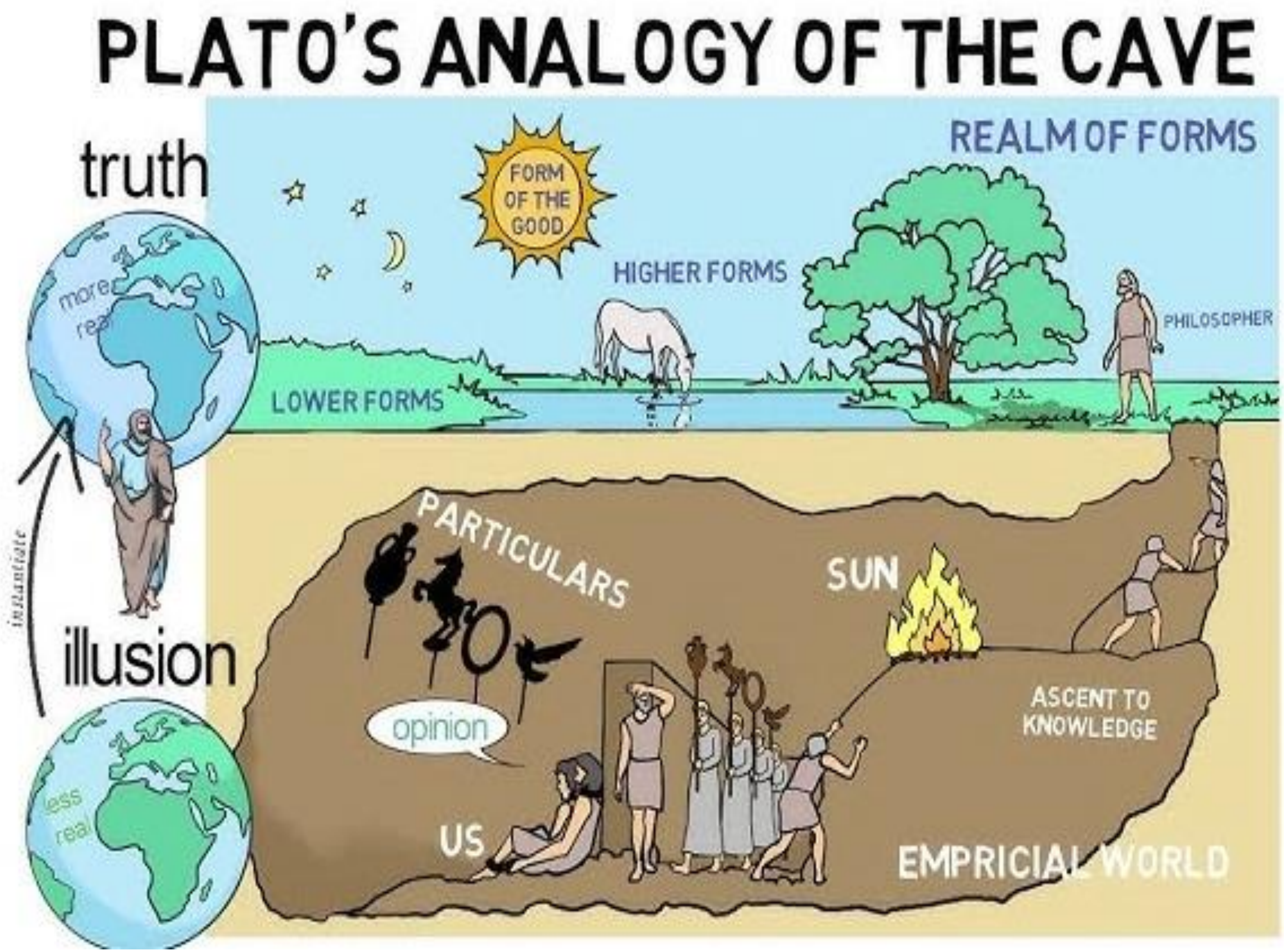
Levin on Platonic space in biology

Mathematicians are already very comfortable with this – the old idea (Plato, Pythagoras, etc.) that there **is a non-physical space of truths which we discover, not invent, and that this space has a structure that enables exploration.** I make the conjecture that this space contains not only low-agency forms like facts about triangles and the truths of number theory, but also a very wide variety of **high-agency patterns that we call kinds of minds. On this view, physical bodies don't create, or even connect to (and thus have) minds – instead, minds are the patterns, with their ingressions into the physical world enabled by the pointers of natural or synthetic bodies. In other words, whenever anything is built – machines, AI's, biobots, hybrots, embryos, etc.** – it acts as an interface to numerous patterns from this space of forms to which guide its form and behavior beyond what any algorithm or material architecture explicitly provides. Michael Levin, Platonic space: where cognitive and morphological patterns come from (besides genetics and environment), March 9, 2025. https://thoughtforms.life/platonic-space-where-cognitive-and-morphological-patterns-come-from-besides-genetics-and-environment/


Platonic forms or patterns of cellular organization and behavior
Ingression
The process is teleological (goal-directed). The goal comes from the Platonic space.
So, we have Aristotelian final, formal, efficient, and material causes.
Egg
Biophysics, biochemistry
Genetics
External environment

Questions for Levin

- What is this “Platonic space?” What metaphysical commitments does Platonic space imply? Could the idea have practical and scientific value without a commitment to full-blown Platonism?
- Do we need it to explain and predict morphogenesis? Aren’t genetics, physics, chemistry, environment, etc. enough?
- How could groups of cells interact with it through their bioelectric network?
- How does the idea of “Platonic space” help advance experimental work and practical applications in synthetic biology, regenerative medicine, and other fields?
- What are some testable hypotheses about “Platonic space” that could be investigated?



Forms are not in space and time

Outside of the material world

Exist independently

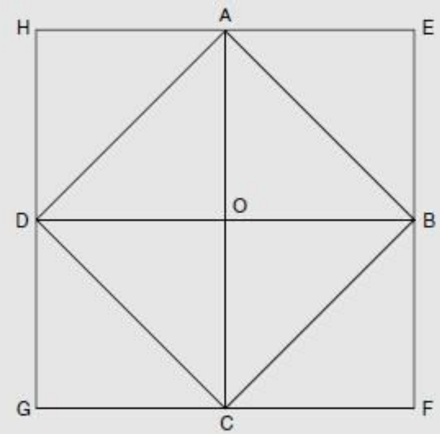
https://supremes-scaling.fandom.com/wiki/Platonic_Forms?file=Plato%27s_Forms.png

Plato’s proof that mathematical knowledge is inborn

- In the *Meno*, Socrates guides a slave boy through a series of steps to show that the area of one square is four times that of the other.
- Since the slave boy had no formal education in geometry, he must have been born with this knowledge; that is, his soul acquired this knowledge when it was in heaven prior to his birth—he recollected the knowledge.
- In the *Phaedo*, Socrates shows that our concept of equality does not come from the physical world because no two things are exactly equal.
- In the *Republic*, Plato developed his theory of knowledge as a form of recollection.

ABEL'S PROOF by Peter Pesic, MIT Press, 2004

Box 1.3
Socrates' construction of the doubled square in the *Meno*



Let the original square be *AEBO*. The slave boy thought that the square on the doubled side *HE* would have twice the area, but realized that in fact that *HEFG* has four times the area of *AEBO*. At Socrates' prompting, he then draws the diagonals *AB, BC, CD, DA* within the square *HEFG*. Each triangle *AOB* = *BOC* = *COD* = *DOA* is exactly half the area of the original square, so all four of these together give the true doubled square *ABCD*.

https://epages.wordpress.com/2018/11/29/platos-meno-the-socratic-dialogue-that-says-we-knew-it-all-the-time/

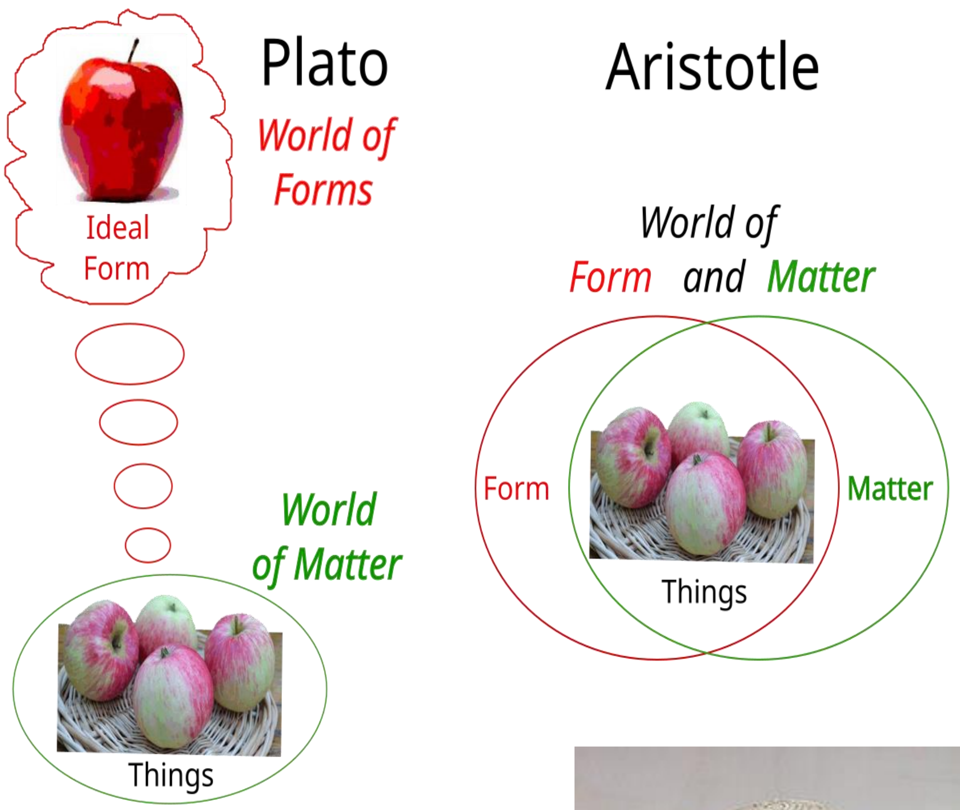
A priori knowledge

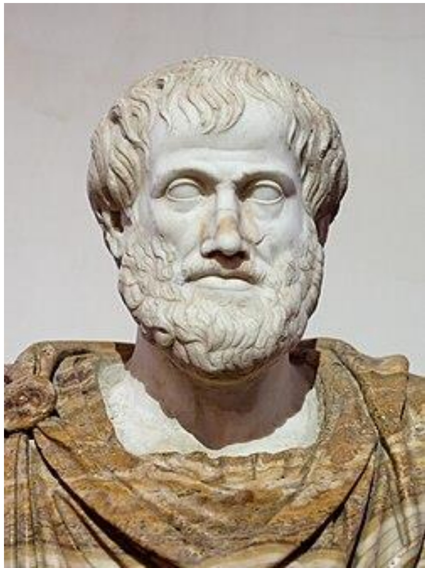
- Plato was demonstrating the existence of what philosophers would later call a priori knowledge (independent of experience).
- Other key proponents of a priori knowledge: Descartes, Kant, GE Moore.
- The is not a very good argument for realism but it calls attention to an important aspect of math—that it seems to be objectively true, independent of experience.

Aristotle

https://commons.wikimedia.org/wiki/File:Platonic_and_Aristotelian_Forms.svg

- Objected to the theory of forms.
- How can we have knowledge of them if knowledge requires physical interaction?
- Forms exist in the world and are realized (or actualized) in physical things.
- We acquire knowledge of the forms empirically, by observation, induction, and abstraction.
- 4 causes—material, efficient, final, and formal.
- Aristotle's ideas dominated philosophy and science for 1500 years.





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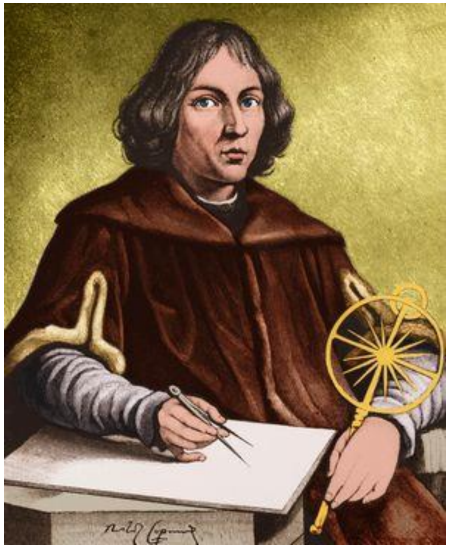
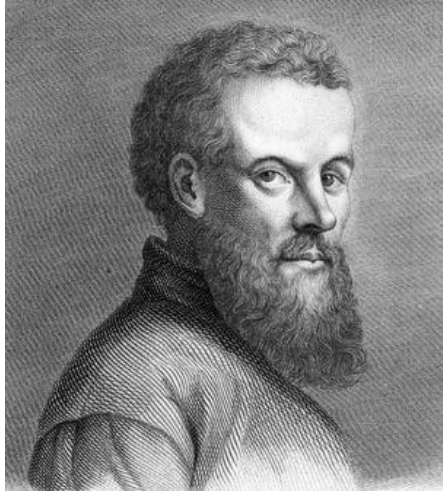
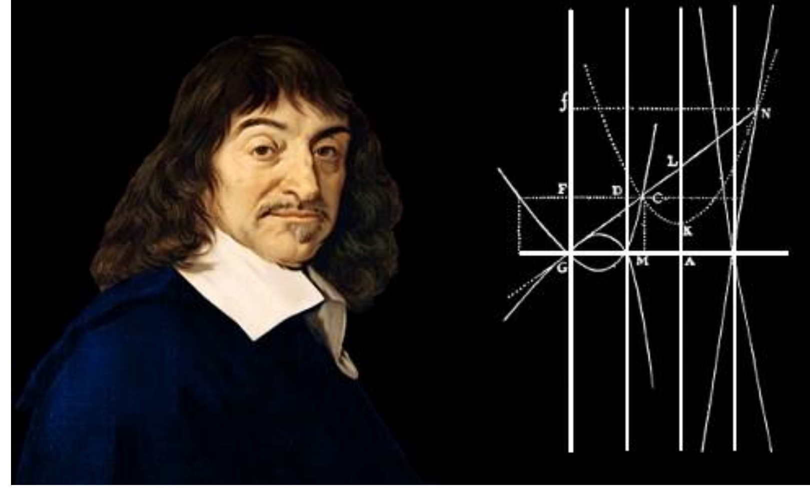
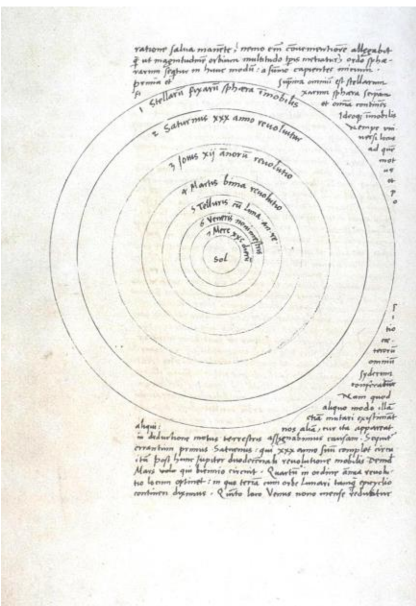
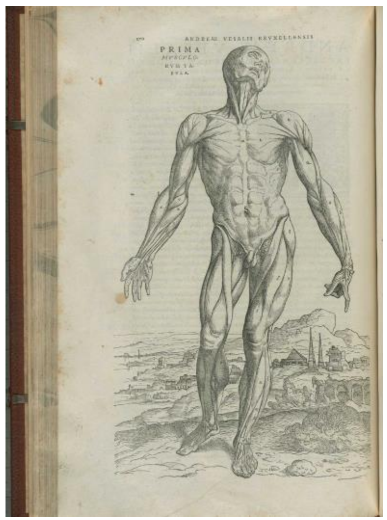
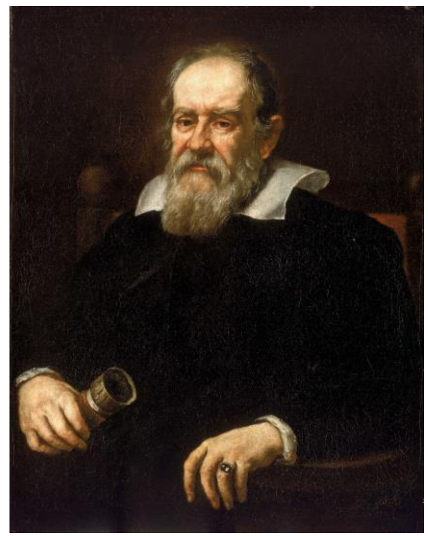










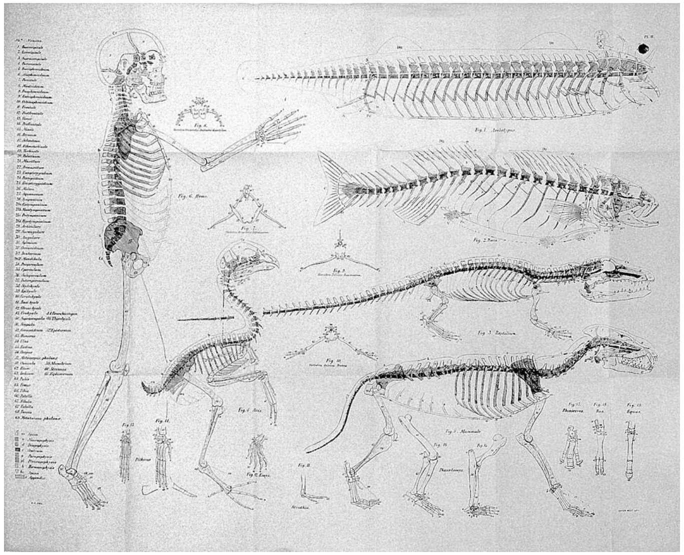


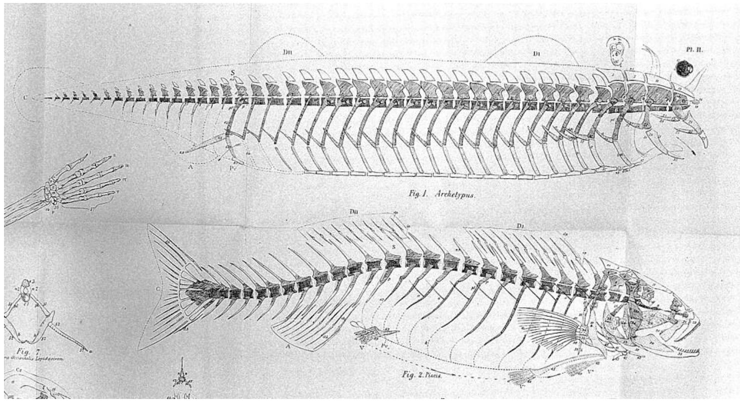
The Scientific Revolution

https://www.britannica.com/biography/Nicolaus-Copernicus
By After Frans Hals - André Hatala [e.a.] (1997) De eeuw van Rembrandt, Bruxelles: Crédit communal de Belgique, ISBN 2-908388-32-4., Public Domain, https://commons.wikimedia.org/w/index.php?curid=2774313
By Justus Sustermans - http://collections.rmg.co.uk/collections/objects/14174, Public Domain, https://commons.wikimedia.org/w/index.php?curid=62614082By Godfrey Kneller - File:Portrait of Sir Isaac Newton, 1689.jpg from https://exhibitions.lib.cam.ac.uk/linesofthought/artifacts, Public Domain, https://commons.wikimedia.org/w/index.php?curid=132521185
https://www.nlm.nih.gov/exhibition/historicalanatomies/Images/1200_pixels/Vesalius_Pg_170.jpg
https://www.britannica.com/biography/Andreas-Vesalius
https://www.famousscientists.org/rene-descartes/

Teleology and Pre-Existing Forms in Biology

- The scientific revolution led to rejection of Aristotelian final causes in physics.
- Search for causal/mechanical explanations of natural phenomena.
- Emphasis on observation, experimentation, mathematics.
- However, biologists continued to search final and formal causes (teleology and forms).
- Naturalists and paleontologists, Jean-Baptiste Lamarck (1744-1829), Georges Cuvier (1769-1832), and Richard Owen (1804-1892) believed that organisms are generated from pre-existing archetypes or forms.

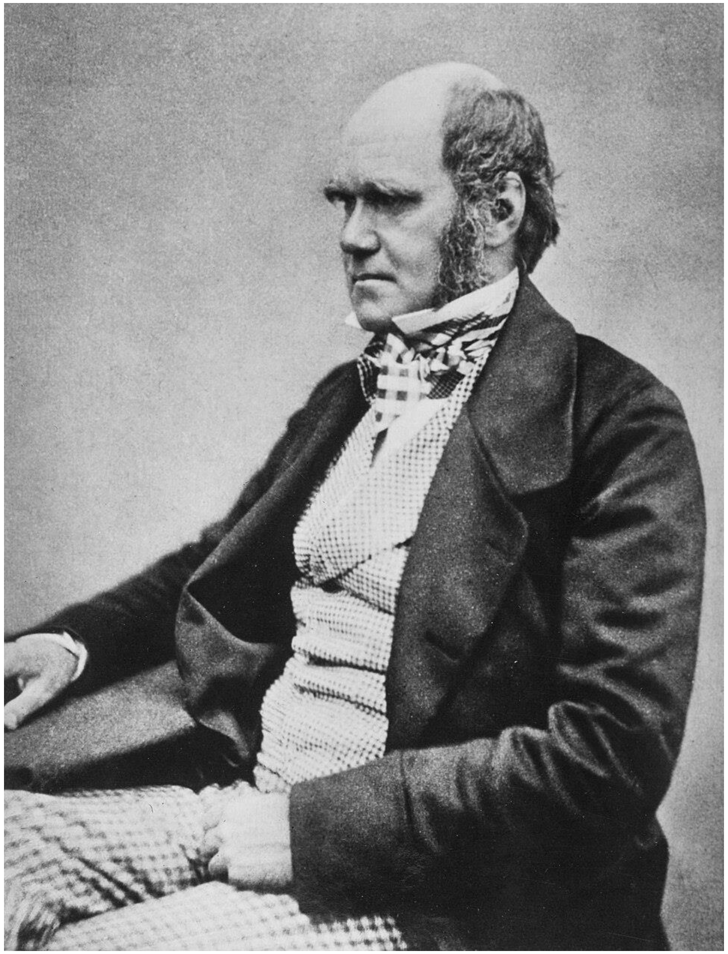




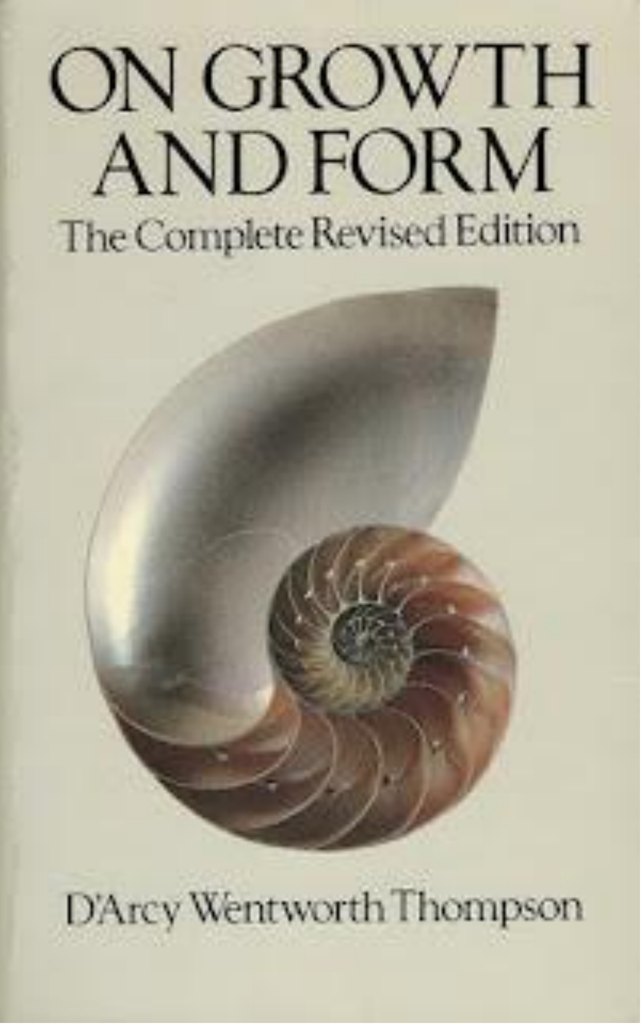
https://commons.wikimedia.org/wiki/File:Sir_Richard_Owen,_On_the_archetype_and_homologies..._Wellcome_L0029110.jpg
https://commons.wikimedia.org/wiki/File:Sir_Richard_Owen,_On_the_archetype_and_homologies..._Wellcome_L0029109.jpg

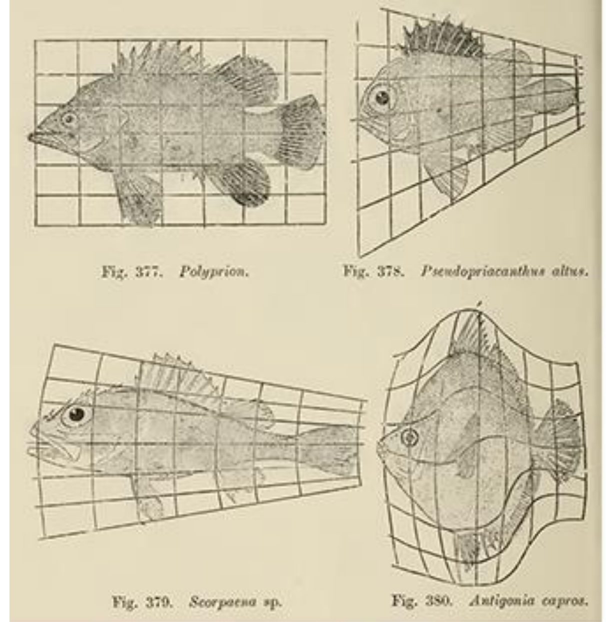
The Darwinian Revolution

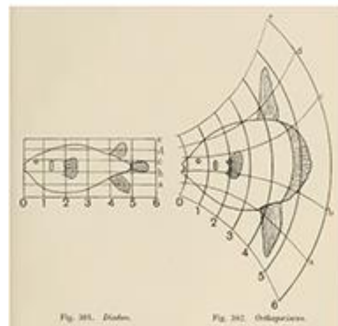
- Charles Darwin's (1809-1882) theory of natural selection depicted the evolution of life as a historical, contingent, process, not dependent on pre-existing forms.
- Species are historical accidents, not natural kinds.
- There are no purposes in nature, divine or otherwise.

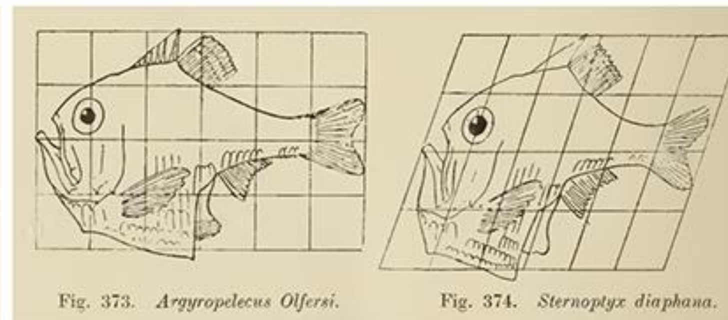


By Charles_Darwin_seated.jpg: Henry Maull (1829–1914) and John Fox (1832–1907) (Maull & Fox) [3]derivative work: Beao - Charles_Darwin_seated.jpg, Public Domain, https://commons.wikimedia.org/w/index.php?curid=11264065









Thompson's studies of proportions

1st edition 1917, 2nd 1942

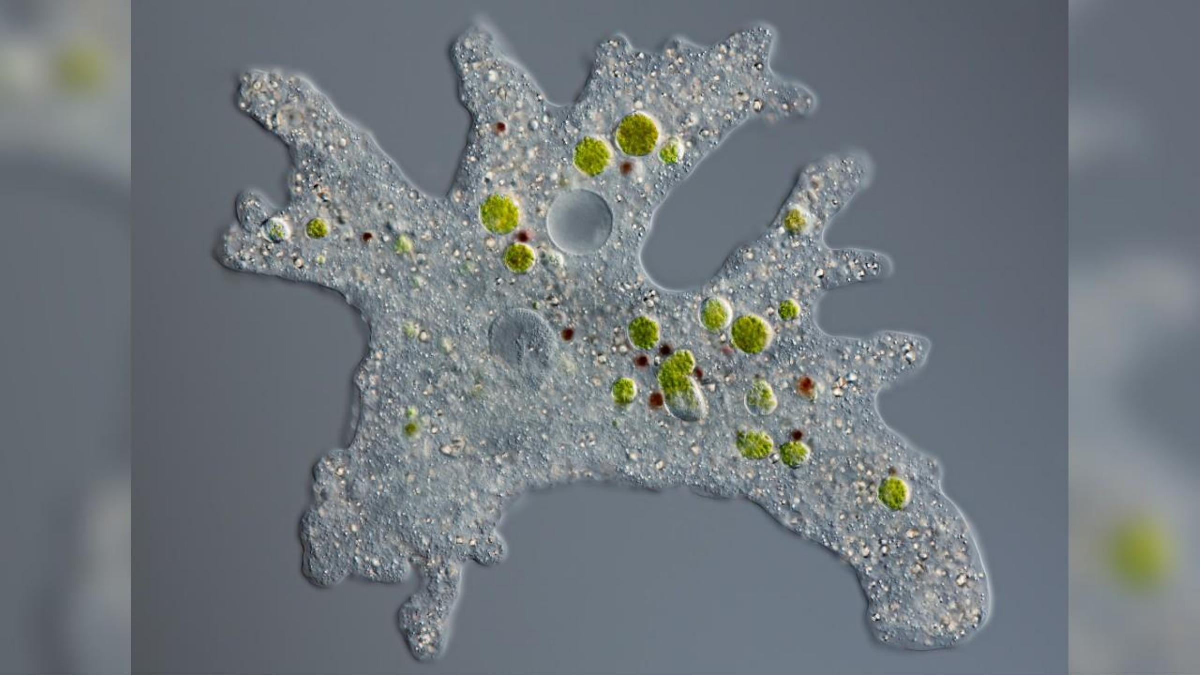
D'Arcy Thompson (1860-1942): life is not entirely an accident; it is constrained and shaped by physics and chemistry and the underlying mathematical forms.

https://writings.stephenwolfram.com/2017/10/are-all-fish-the-same-shape-if-you-stretch-them-the-victorian-tale-of-on-growth-and-form/

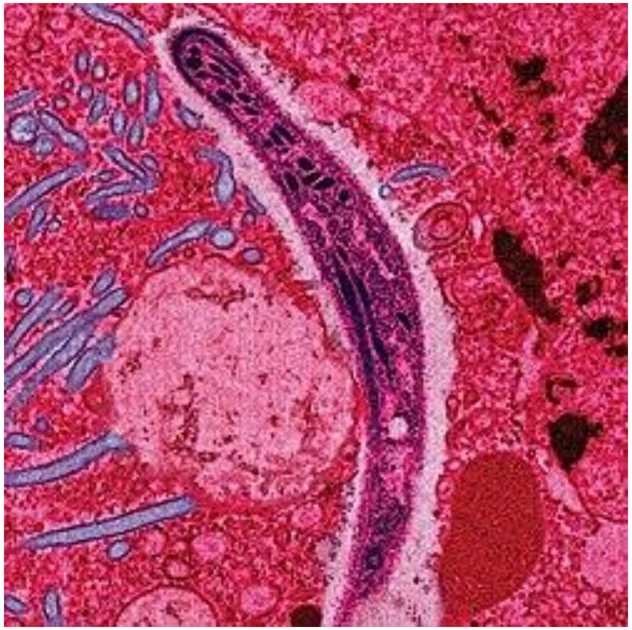
https://www.livescience.com/54281-amoeba-definition.html

Cell and tissue, shell and bone, leaf and flower, are so many portions of matter, and it is in obedience to the laws of physics that their particles have been moved, moulded and conformed. They are no exceptions to the rule that God always geometrizes. Their problems of form are in the first instance mathematical problems, their problems of growth are essentially physical problems, and the morphologist is, ipso facto, a student of physical science. P. 10.

D Arcy Thompson (1860-1948)



"The shape and movement of cells are determined by gravity, air pressure, viscosity of liquid, flow dynamics, surface tension of water, diffusion, electrical and chemical forces, properties of lipid bilayer membrane, thermodynamics, pp. 342-73)."



"When a plasmodium…creeps toward a damp spot or warm spot, or toward substances which happen to be nutritious…we are dealing with phenomena which too often are ascribed to 'purposeful' action or adaption, but every one of which can be paralleled by ordinary phenomena of surface tension, p. 361."

Davies, J.A. D'Arcy Thompson and Synthetic Biology—Then and Now. *Biol Theory* **20**, 92–104 (2025). https://doi.org/10.1007/s13752-024-00476-2

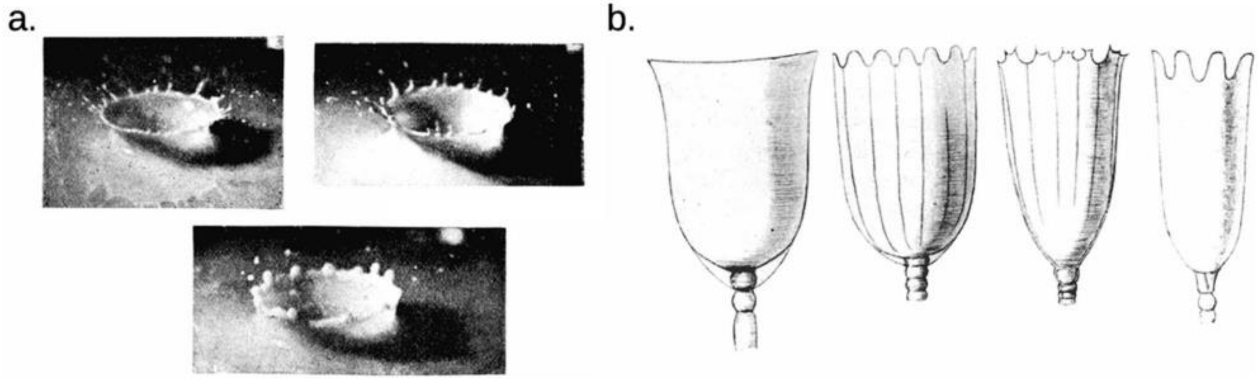


Fig. 5 Two images from *On Growth and Form*, (**a**) showing the shapes created by milk splashes, and (**b**) showing the in-some-ways-similar shaped calycles of *Campanularia* species (Thompson 1917, pp. 389, 391).

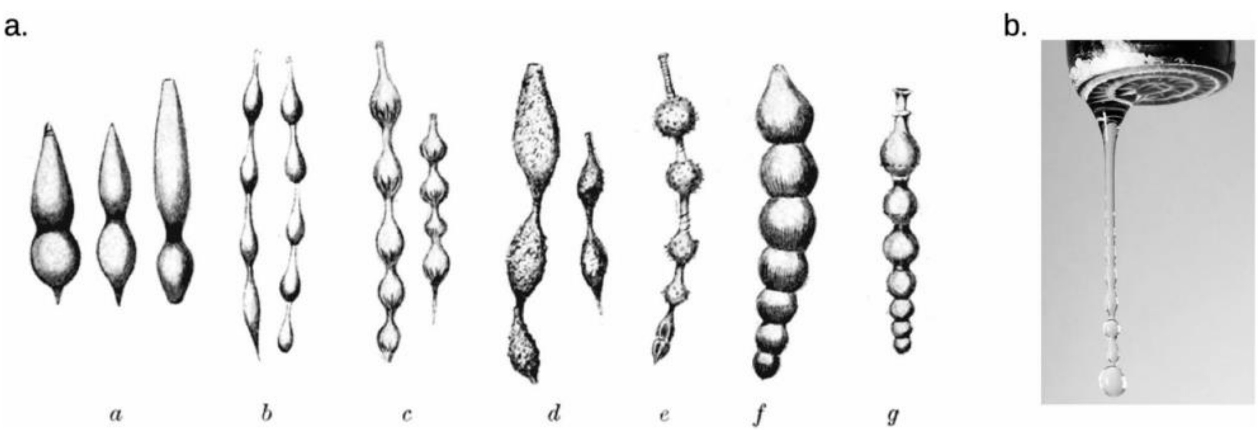
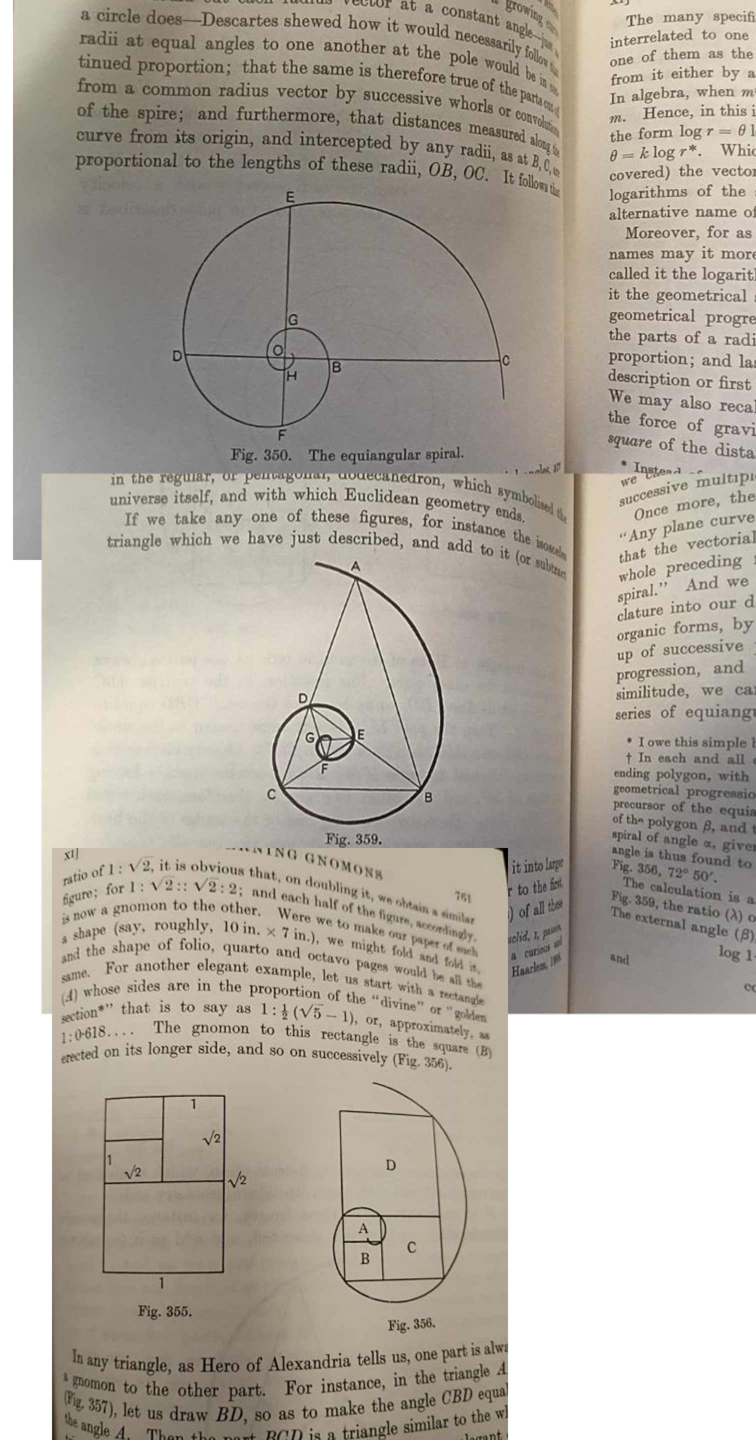


Fig. 6 The resemblance of foraminiferan forms to a column of water breaking up under surface tension (the Plateau-Rayleigh instability). (**a**) shows a figure from *On Growth and Form*, depicting various species of the foraminifera (Thompson 1917, p. 422). (**b**) is a modern photograph of the breakup of a column of water from a tap. Image credit for (**b**): Dschwen. CC-SA-2.5 via Wikimedia Commons, https://commons.wikimedia.org/wiki/File:Dripping_faucet_2.jpg.

Thompson drew analogies between physical processes and structures and biological ones, claiming that similar mathematical principles govern both.

Thompson on Spirals

- Of the spiral forms we have mentioned, every one…is an example of a remarkable curve known as the equiangular or logarithmic spiral..(751)
- In the growth of a shell, we can conceive of no simpler law than this, namely, that is shall widen and lengthen in the same unvarying proportions…the shell, like the creature within it, grows in size but does not change its shape. (757)
- What actually grows is merely the lip of an orifice, where there is produced a ring of solid material…and this generating curve grows in magnitude without changing form (848)
- We have then a certain definite type or group of forms, mathematically isomorphous (848)

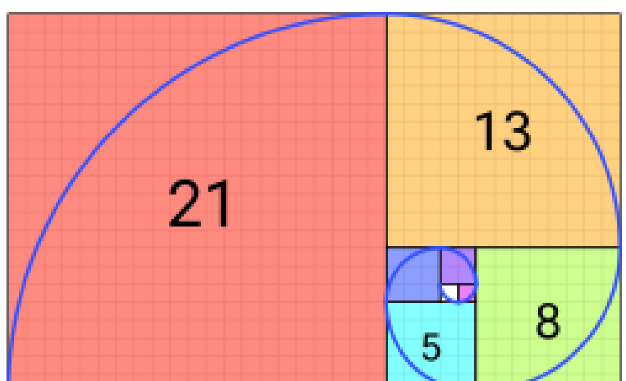


Leonardo Fibonacci
(1170-1250, Pisa)

Fibonacci series

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987…

$F(n) = F(n-1) + F(n-2)$, where $F(n)$ represents the nth Fibonacci number, and the sequence typically starts with $F(0) = 0$ and $F(1) = 1$.

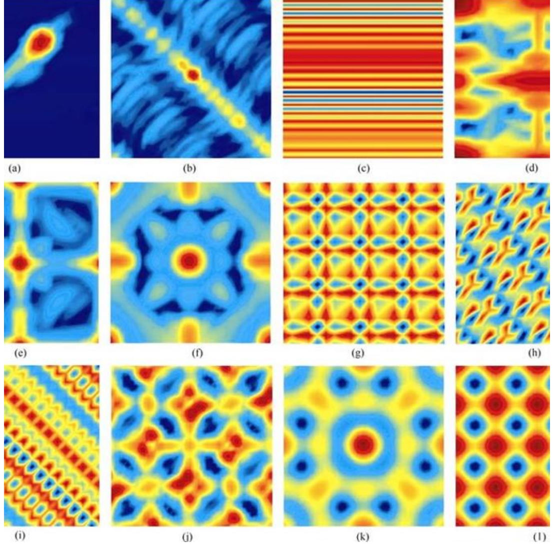


Thompson on Biology, Physics, Math

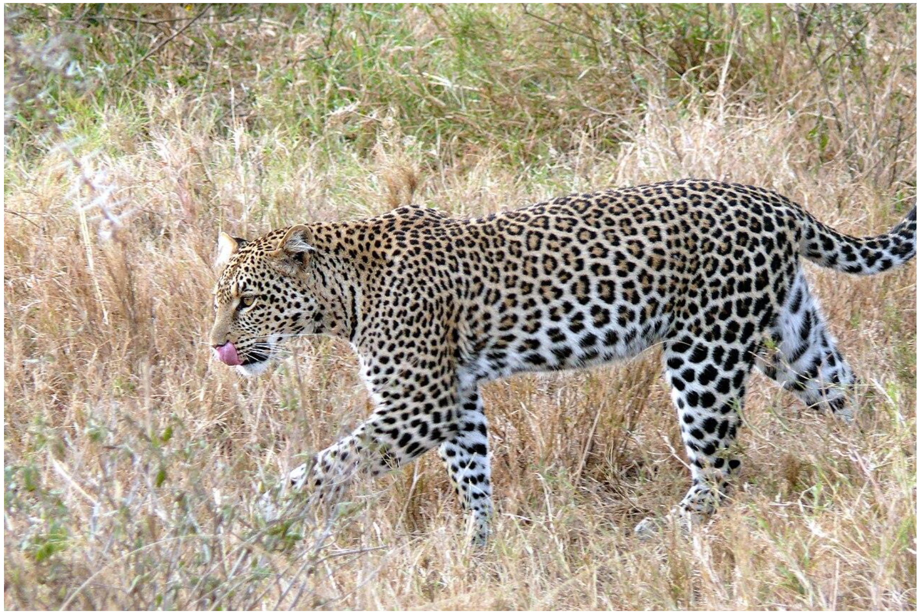
- Thompson's view is that math underlies biological form, but that the math is embodied in the laws of physics and chemistry.
- Physics, chemistry, geometry explain how you get specific forms in biology.
- **Math constrains physics and chemistry; complex patterns emerge from the bottom-up.**

Alan Turing (1912-1954)

- In “The Chemical Basis of Morphogenesis” 1952; Phil. Trans. R. Soc. Lond. B 237: 37–72),” Alan Turing developed a mathematical model to demonstrate how chemicals, which he called morphogens, can produce patterns of phenotypes due to the breaking of symmetries in the embryo. Because of this, there is differential diffusion of and reaction to morphogens, with differential downstream effects.
- Math and chemistry explain the patterns; no need to appeal to any internal representation of the patterns**; patterns emerge.**



Xiao, J., Li, H., Yang, J. et al. from “Chaotic Turing Pattern Formation in Spatiotemporal Systems.” Front. Phys. China



By Haplochromis - Own work, CC BY-SA 3.0, https://commons.wikimedia.org/w/index.php?curid=7460918

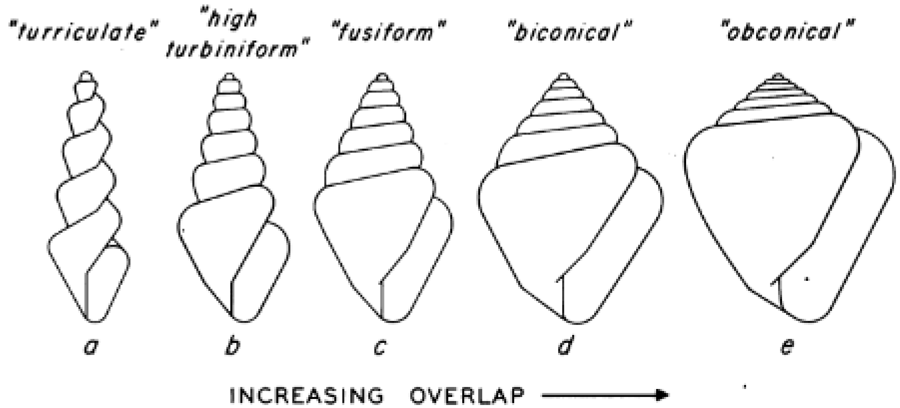


Fig. 1.—Effects on gastropod morphology of changes in amount of whorl overlap.

about a fixed axis (the axis of coiling). In forms such as the one in Figure 1*a*, the shape of this curve is observable: it is nearly equivalent to the aperture or to a cross section of any whorl. But as the amount of whorl overlap increases, part of the outline of the generating curve becomes obscure. In fact, the gastropod represented in Figure 1*e* does not utilize the entire generating curve; rather, only the outer part is actually deposited by the organism. The aperture in this case is enclosed between the inner surface of the body whorl and the outer surface of the previous whorl and reflects the generating curve only along its outer margin.

In Figure 2, the effects of changing the rate of whorl expansion are shown by

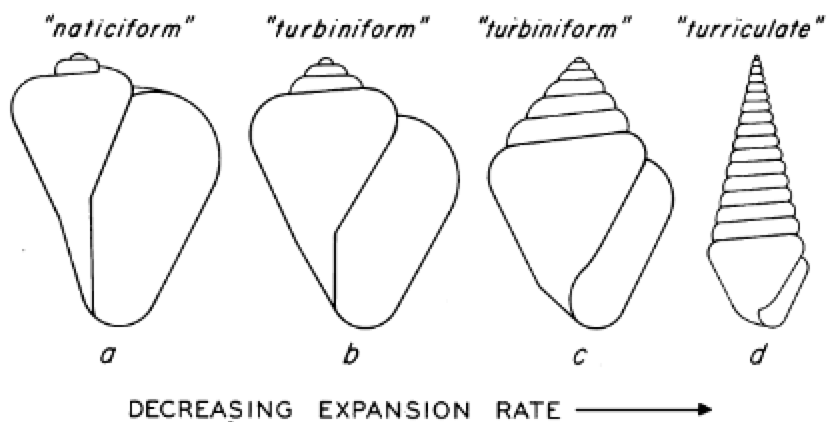
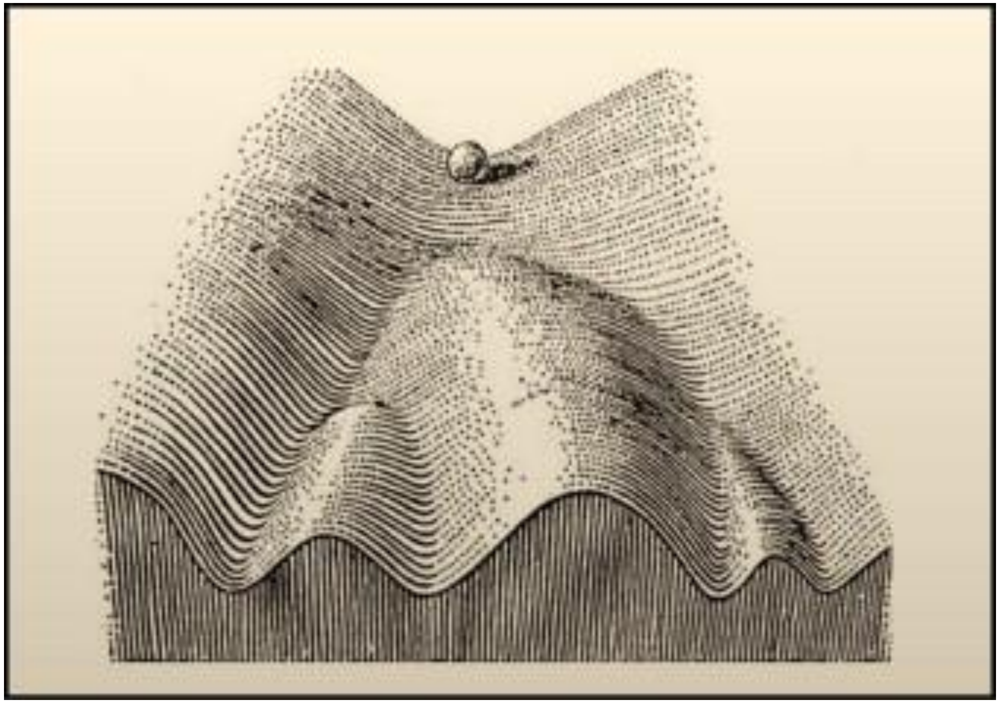


Fig. 2.—Effects on gastropod morphology of changes in rate of whorl expansion.

Raup (1933-2015) built upon Thompson's work with the idea of a morphospace as possible morphologies organisms can exhibit. He viewed it as a constraint on evolution, so that different species of shells, for example, would occupy different places in the morphospace. The constraint was due the effects of physics, chemistry, and geometry on evolution. However, organisms do not reach out, internally represent, or navigate through the morphospace. They passively move along it, pushed in different directions by environmental factors.

Raup, David M. "The geometry of coiling in gastropods." Proceedings of the National Academy of Sciences 47.4 (1961): 602-609.

Conrad Waddington Epigenetic landscape



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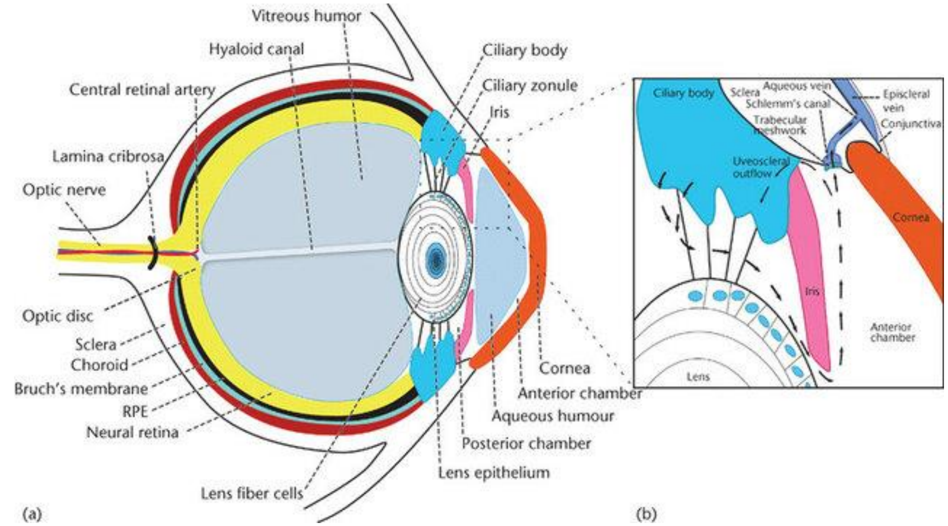
Conrad Waddington's (1905-1975) idea of the epigenetic landscape represents different pathways cells can take as they differentiate. The pathways are stable forms. Genetic and epigenetic, and external environmental influences probabilistically nudge the cell in different directions. This idea was also extended to the behavior of groups of cells. Cells do not reach out, internally represent, or navigate through the landscape. They passively move along it, pushed in different directions.

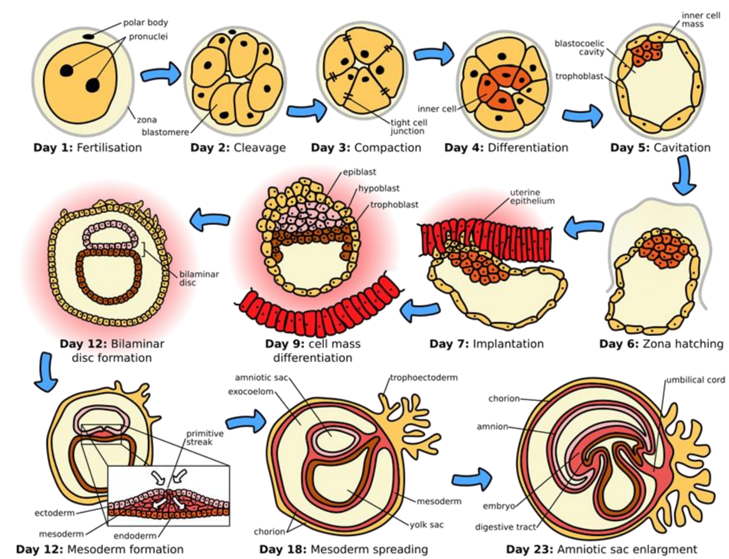
The Strategy of the Genes (1957)

Limitations of emergence

- Complex, specialized structures or processes that seem to involve much more than bending, folding, symmetry breaking, diffusion of morphogens, surface tension, homeobox genes, and so on; for example, eyes.
- Some key information seems to be missing that is needed to explain and predict how these systems are generated—**where to go in morphospace, and when to stop.**
- **These patterns don't just emerge.**



https://www.researchgate.net/publication/277708055_Eye_Anatomy/figures?lo=1



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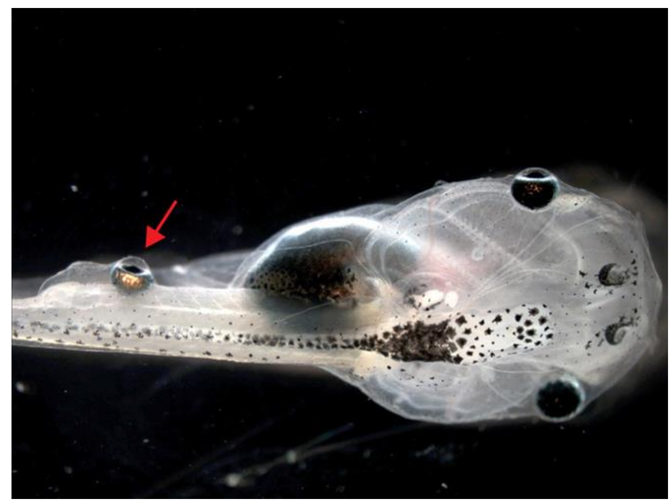
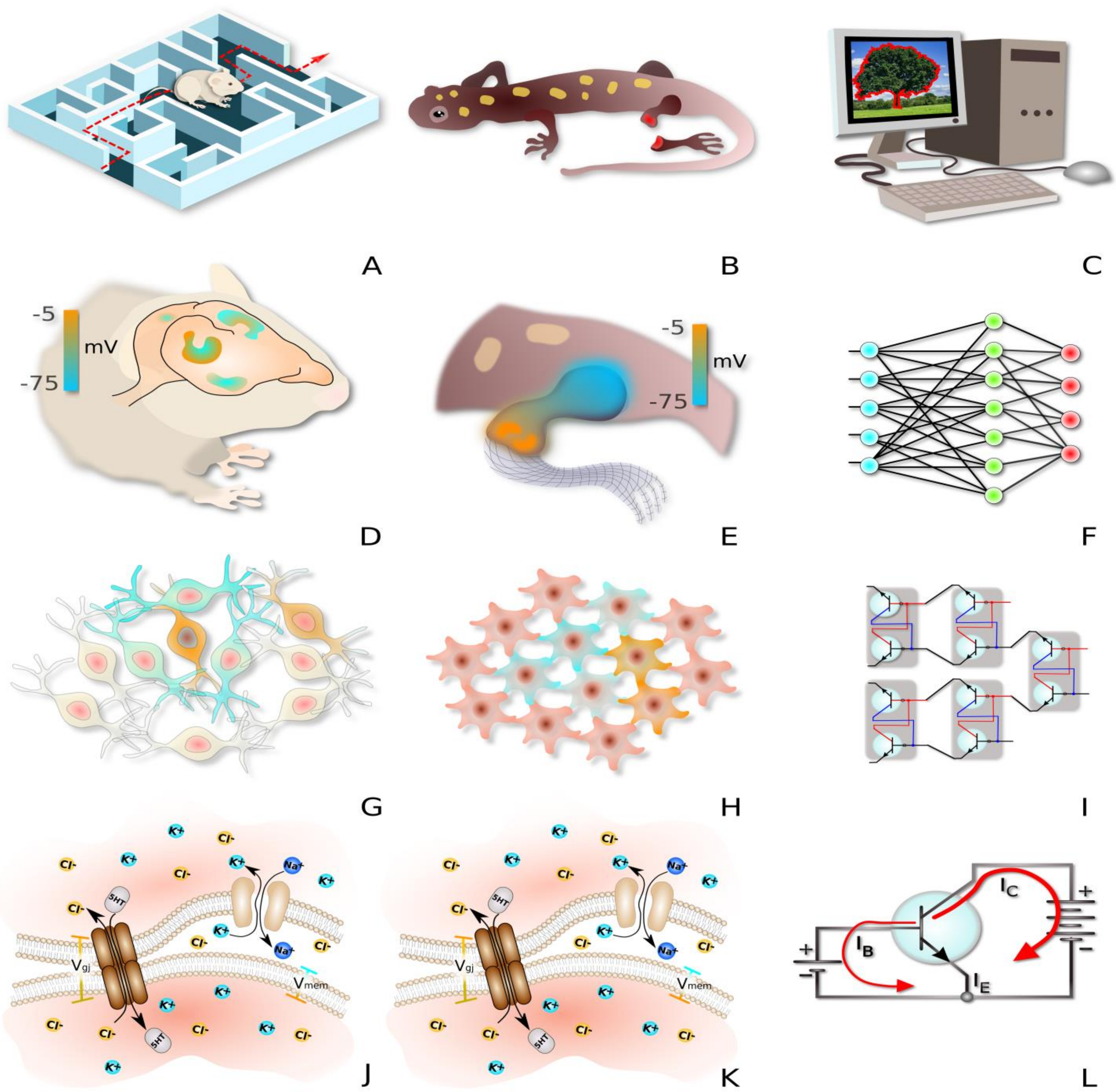
Levin's ground-breaking experiments that challenge the emergence paradigm



https://www.science.org/content/article/franken-tadpoles-see-eyes-their-backs https://pioneerworks.org/broadcast/xenobots-2-claire-evans
https://www.researchgate.net/publication/277708055_Eye_Anatomy/figures?lo=1
https://www.newscientist.com/article/2132148-bioelectric-tweak-makes-flatworms-grow-a-head-instead-of-a-tail/
https://www.quantamagazine.org/cells-form-into-xenobots-on-their-own-20210331/ https://www.sci.news/biology/planarian-flatworms-grow-heads-brains-other-flatworm-species-03462.html

Levin provides evidence for a mentalistic approach to goal-directedness in biology

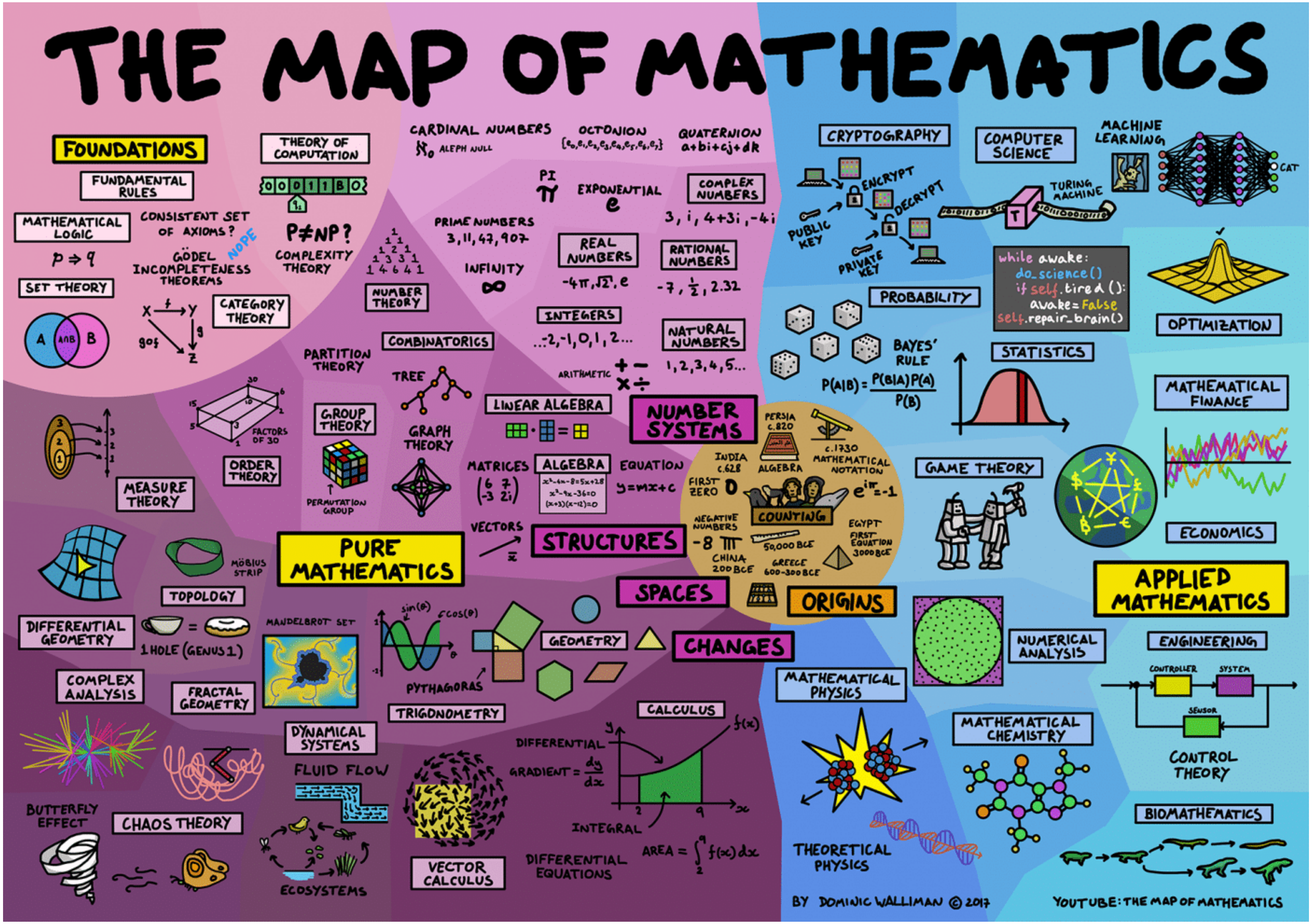
- Complex structures do not just (always) emerge from the underlying physics and chemistry.
- Sometimes—perhaps often—they are the result of goal-directed processes; that is, organisms are seeking to realize goals.
- Goals are internally represented in bioelectric networks (and other mechanisms).
- The goal tells the organism what structure to form and when it has formed it (when it has reached the goal).
- Organisms are intelligent agents that effectively pursue goals.
- There are degrees of intelligence corresponding to degrees of goal-directedness.
- All organisms are collective intelligences.



Slide courtesy of Michael Levin; Drawing by Alexis Pietak

Mathematics

- Cells are acting on information that does not come directly from genetics, physics, or chemistry.
- Where does this information come from?
- How do cells represent it, respond to it, and use it?
- Levin: the information comes from the Platonic space and cells represent it and use it via bioelectric networks.
- The information is represented in the bioelectric network.
- We should be able to use math to study these forms and predict how cells will respond under different conditions.
- This is all very interesting, but as we have seen, it creates some philosophical and scientific issues that must be worked out.


THE MAP OF MATHEMATICS
FOUNDATIONS
FUNDAMENTAL RULES
MATHEMATICAL LOGIC
CONSISTENT SET OF AXIOMS?
NOPE
GÖDEL INCOMPLETENESS THEOREMS
SET THEORY
THEORY OF COMPUTATION
P≠NP?
COMPLEXITY THEORY
CATEGORY THEORY
CARDINAL NUMBERS
ALEPH NULL
OCTONION
QUATERNION
a+bi+cj+dk
PI
EXPONENTIAL
COMPLEX NUMBERS
3, i, 4+3i, -4i
PRIME NUMBERS
3, 11, 47, 907
INFINITY
REAL NUMBERS
-4π, √2, e
RATIONAL NUMBERS
-7, 1/2, 2.32
NUMBER THEORY
INTEGERS
..-2,-1,0,1,2...
NATURAL NUMBERS
1,2,3,4,5...
ARITHMETIC
COMBINATORICS
PARTITION THEORY
TREE
MEASURE THEORY
FACTORS OF 30
ORDER THEORY
GROUP THEORY
PERMUTATION GROUP
GRAPH THEORY
LINEAR ALGEBRA
MATRICES
ALGEBRA
EQUATION
y=mx+c
NUMBER SYSTEMS
VECTORS
STRUCTURES
PURE MATHEMATICS
TOPOLOGY
MÖBIUS STRIP
1 HOLE (GENUS 1)
DIFFERENTIAL GEOMETRY
COMPLEX ANALYSIS
FRACTAL GEOMETRY
MANDELBROT SET
sin(θ)
cos(θ)
PYTHAGORAS
GEOMETRY
SPACES
CHANGES
TRIGONOMETRY
CALCULUS
DYNAMICAL SYSTEMS
FLUID FLOW
DIFFERENTIAL
GRADIENT = dy/dx
INTEGRAL
AREA = ∫ f(x) dx
DIFFERENTIAL EQUATIONS
VECTOR CALCULUS
BUTTERFLY EFFECT
CHAOS THEORY
ECOSYSTEMS
ORIGINS
PERSIA c.820 ALGEBRA
INDIA c.628 FIRST ZERO
c.1730 MATHEMATICAL NOTATION
NEGATIVE NUMBERS
CHINA 200 BCE
COUNTING
50,000 BCE
GREECE 600-300 BCE
EGYPT FIRST EQUATION 3000 BCE
CRYPTOGRAPHY
ENCRYPT
DECRYPT
PUBLIC KEY
PRIVATE KEY
COMPUTER SCIENCE
TURING MACHINE
MACHINE LEARNING
CAT
while awake:
do_science()
if self.tired():
awake = False
self.repair_brain()
OPTIMIZATION
PROBABILITY
BAYES' RULE
P(A|B) = P(B|A)P(A) / P(B)
STATISTICS
MATHEMATICAL FINANCE
GAME THEORY
ECONOMICS
APPLIED MATHEMATICS
NUMERICAL ANALYSIS
ENGINEERING
CONTROLLER
SYSTEM
SENSOR
CONTROL THEORY
MATHEMATICAL PHYSICS
MATHEMATICAL CHEMISTRY
THEORETICAL PHYSICS
BIOMATHEMATICS
BY DOMINIC WALLIMAN © 2017
YOUTUBE: THE MAP OF MATHEMATICS

Mathematics is Philosophically Perplexing

Mathematics seems to be about objective facts.

Normally, when we claim that a statement is true, we assume that the thing it refers to exists and we assume that we can causally interact with things that exist, for example:

“The Statue of Liberty is in New York Harbor.”

We can go to New York Harbor an verify this statement. We can even touch the Statue of Liberty. But what about this one:

“2 is an even number.”

2 is an abstract object with no particular location in space and time. We can’t reach out and touch the number 2. So maybe mathematical statements are like this:

“Sherlock Holmes is a brilliant detective.”

Sherlock Holmes does not exist but we can still say that statement is true because it is about something that is true with respect to our minds and/or language.

Philosophy of Mathematics—Basic Positions

Realism "math is discovered, not invented"

- **Mathematics is about objective facts (or true statements) that are independent of mind, language, or culture.**
- **Supernaturalism** (Plato, Frege, Gödel)**:** Mathematical objects exist in a transcendent realm outside of the natural world.
- **Naturalism** (Quine, Aristotle, Resnik, Maddy): Mathematical objects are part of the natural world.

Non-Realism "math is a human invention"

- **Mathematics is <u>not</u> about objective facts (or true statements) that are independent of mind, language, or culture.**
- **Intuitionism** (Kant, Brouwer; Dodig-Crnkovic*): Mathematical objects are constructed by the mind and exist only in the mind.
- **Formalism** (Hilbert): Mathematical objects are symbols in a formal system that do not refer to anything.

*doi: 10.20944/preprints202512.0033.v1

An Indispensability Argument for Realism

1. **Mathematics plays an indispensable role in formulating, testing, and applying theories, hypotheses, and models used science, engineering, medicine, and other epistemic practices that have been highly successful in explaining, predicting, and controlling events and processes in the physical world.***

2. **The best explanation of the role of mathematics in these successful practices is that the physical world has a mathematical structure. This is Wigner's view.****

3. **Therefore, by IBE, we are justified in believing that the world has a mathematical structure.**

4. Since we are justified in believing that the world has a mathematical structure, we are also justified in believing that 1) the truth of the mathematical statements used in science, engineering, etc. is independent of mind, language, or culture and 2) the mathematical objects referred to in science, engineering, etc. exist independently. If the mathematical statements and ontologies used in these practices were only mental fictions or linguistic tools, it would be extremely unlikely that calculations based on them would work as well as they do.

*Quine, Putnam, others have formulated different versions of this premise. Colyvan M. 2023. Indispensibility argument in the philosophy of mathematics. https://plato.stanford.edu/entries/mathphil-indis/#ExplVersArgu. *Wigner E. 1960. The unreasonable effectiveness of mathematics. Communications in Pure and Applied Mathematics 13 (1): 1-14.

Inference to the Best Explanation (abduction)

A form of non-deductive reasoning.

Evidence A [for example, I hear howling at night in North Carolina]

H is the best explanation of A [the best explanation is that the howling comes from coyotes]

Therefore, H is probably true.

> ***or Therefore, we are justified in believing H is true [weaker, more pragmatic]***

Issues:

What is the best explanation? What are criteria for a good explanation?

Is this legitimate reasoning? Isn't it biased by our background theories, etc. that tell us that an explanation is good or not?

Maybe IBE is a good starting point but as we learn more use other inductive methods, such as statistical hypothesis testing and Bayesian inference.

https://plato.stanford.edu/entries/abduction/

Limitations and Weaknesses of this Argument

- **Only applies to the math that is indispensable for science, engineering, etc.** Reply: perhaps, but that may be most of math and math may have a unity such that you get the whole package with the argument.
- **Realism is not the best explanation for the usefulness of mathematics in science, engineering, etc.** Reply: what's a better explanation? Realism may be the only explanation!
- **IBE is controversial; we need a more rigorous form of argument.** Reply: Perhaps—but show me one. You won't get a deductive argument, and other types of arguments have their problems too (e.g., biases priors in Bayesian reasoning).

Supernaturalism vs. Naturalism

- **Dualistic metaphysics is not compatible with the modern scientific worldview.**
  - Reply: Science can survive dualism. We need it in psychology too.
- **How can we have mathematical knowledge if this requires causal interaction with what is known?**
  - Reply: We can use intuition.

Isn't intuition biased? Subjective?

- **How can mathematical objects be "in" the physical world if they are abstract?**
  - Reply: the natural world is mathematically structured; we use abstract concepts, like numbers, sets, and shapes to represent this structure.
- **How do we acquire knowledge of such things?**
  - Reply: We can acquire knowledge of axioms and definitions by abstraction, pattern matching, epistemological holism.

But what about knowledge of infinity, higher order dimensional spaces?

Which approach provides a better explanation of morphogenesis and more fruitfully guides hypothesis testing, theory construction, and experimentation?

My view: <u>Structuralism</u>. Math is the superstructure of the natural world; the canvas on which natural phenomena occur.

Resnik MD. 1997. Mathematics as a Science of Patterns. New York, NY: Oxford University Press.
Tegmark, M. 2008. The Mathematical Universe. Found Phys 38, 101–150 (2008). https://doi.org/10.1007/s10701-007-9186-9

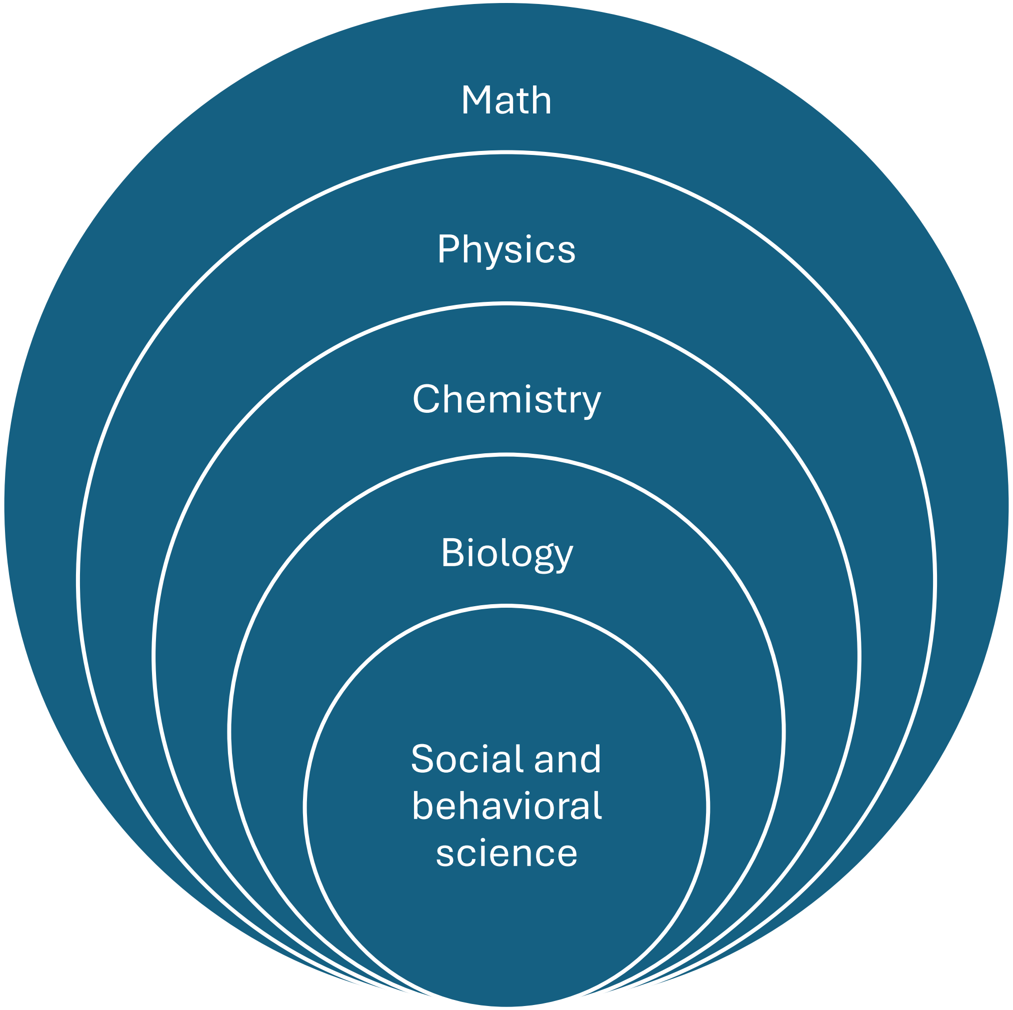
Key distinction:

Mathematical structure of the natural world

Vs.

Our representation of that structure in language or mental categories

For example, we can say that the world has a numerical structure without equating that structure with any particular number system (e.g., binary, Arabic, Roman). Choices of language and concepts are partly pragmatic, based on our needs, interests, and computational abilities.



Mathematics Structures the World: Constraining and Enabling

- Mathematics delineates structured spaces for interactions, events, and processes that happen in the natural world.
- Think of a dice game with 2, 6-sided dice. Mathematics **<u>enables</u>** the game by delineating a structured space of possibilities for the game. You can roll a total of 2,3,....12.
- Mathematics also **<u>constrains</u>** the game by delineating events that are not possible. You cannot get a total of 0,1, 13, etc.
- Note: you could have letters on the dice, e.g., a, b, c, d, e, f and math would still operate in the same way in terms of possible combinations.

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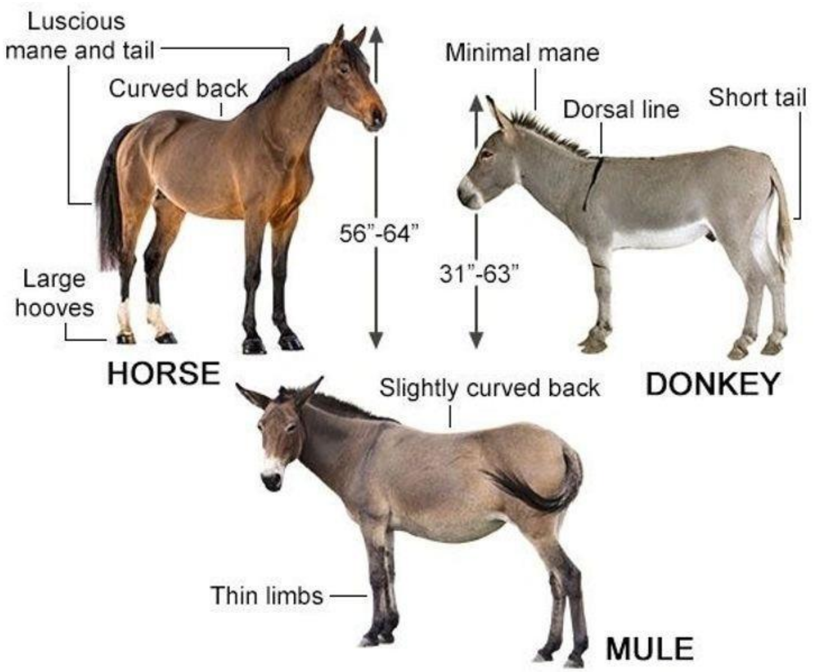
Mathematical Explanations (not causal, but structural)

https://en.wikipedia.org/wiki/Brood_X

- **Physics—symmetry** explains conservation laws, transformations, particle/anti-particles, etc.
- **Periodical cicadas** offer another example of the role of how mathematics can create a space of possibilities for biological phenomena. Periodical cicadas have cycles of 13 or 17 years, depending on the species. Gould (1977) proposed that the reason cicadas have 13- or 17-year cycles is that these cycles are prime numbers, which are more difficult for predator species to synchronize with over time than non-primed numbered cycles. If periodical cicadas had a 12-year cycle, for example, then a predator with life cycle of 3 years would be able to prey on them 4 times in a 12-year span. Thus, mathematics (number theory) creates a structured space of possibilities for life cycles which have different effects on the fitness of organisms.

For another example, consider sexual reproduction. Most adult organisms that reproduce sexually are diploid, with one set of chromosomes from each parent. To reproduce, adult organisms exchange haploid gametes (sperm and eggs) which fuse to create diploid zygotes. Haploid gametes are produced in gonadal cells by meiosis, a process in which diploid cells evenly divide their chromosome pairs between two cells. For example, an adult horse has 64 chromosomes, which are divided in half to 32 during meiosis. Mules are infertile because they have an odd number of chromosomes—63; 32 from their mother (a horse) and 31 from their father (a donkey) (The Tech Interactive 2007). Thus, mathematical facts about odd vs. even numbers create a structured space of possibilities for sexual reproduction but also impose constraints on reproduction.



https://thebritishmulesociety.com/All-About-Mules/

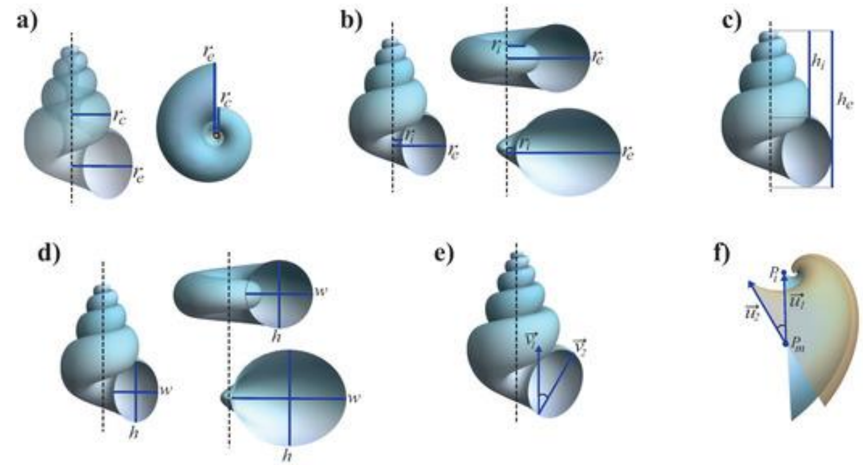
Thompson (1942) thought honeycomb construction was constrained by physical forces on the wax and mathematical principles underlying these laws (geometry of efficient filling of space).

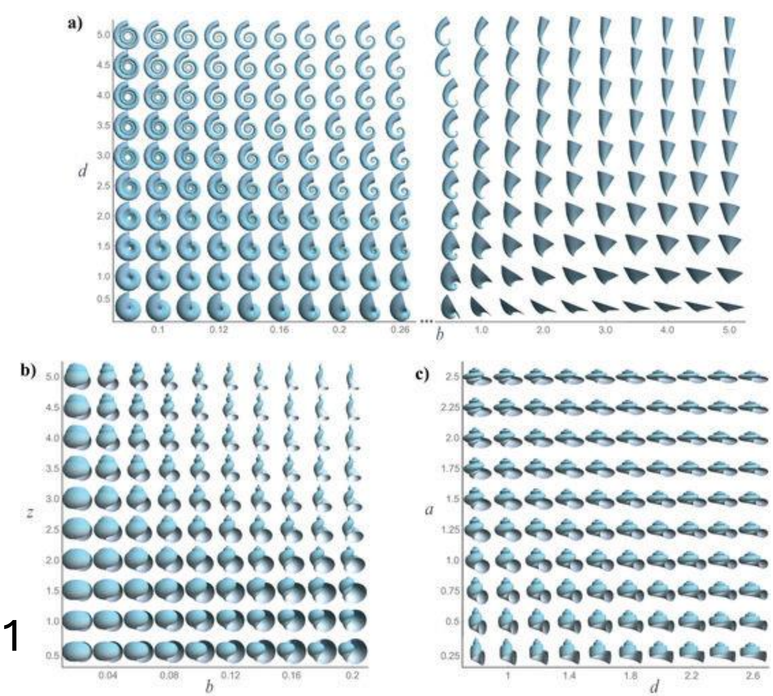
But there’s more to it than this. The hexagonal shape only emerges when cells are aligned in a certain way and bees decide how to align cells. When the bees make a queen cell, you get a different shape. So, math creates a space of possible shapes that bees and natural selection can explore over time.

https://www.google.com/imgres?imgurl=https%3A%2F%2Fpreview.redd.it%2Fqueen-cell-v0-cdvdupp9lpec1.jpg%3Fwidth%3D640%26crop%3Dsmart%26auto%3Dwebp%26s%3Deb0e3d6befbe857f1e653e94b6d604e4bce2b0b3&tbnid=9hrr63X7bp4HlM&vet=1&imgrefurl=https%3A%2F%2Fwww.reddit.com%2Fr%2FBeekeeping%2Fcomments%2F1ab9e8x%2Fqueen_cell%2F&docid=h7IkUcojnZT1VM&w=640&h=853&source=sh%2Fx%2Fim%2Fm1%2F2&kgs=5aea6f7a0354adf1&shem=isst%2Csdl1pl

Thompson vs. Levin on math and morphogenesis

- Thompson emphasized mathematics as a constraint on morphogenesis that operates through the laws of physics and chemistry. Form emerges from bottom up.
- Levin is proposing that mathematics creates a space of possible forms ("Platonic space") that organisms can internally represent and use to guide morphogenesis. Form can come from top-down.





https://doi.org/10.3390/d15030431

Thompson vs. Levin

https://www.renegadekitchen.com/blog/rock-candy-crystallization

How do organisms use mathematical forms as morphological goals?

- Human beings can mentally represent mathematical objects and use this representation to guide behavior.
- There needs to be feedback between the person and the world so they can know when they have reached their goal, such as cutting a piece of wood to a specified length.

For example: suppose this person has tasked with cutting a piece of the board of a specific size and shape. To do this, they need to mentally represent the board and the relationships between its dimensions, which requires math.

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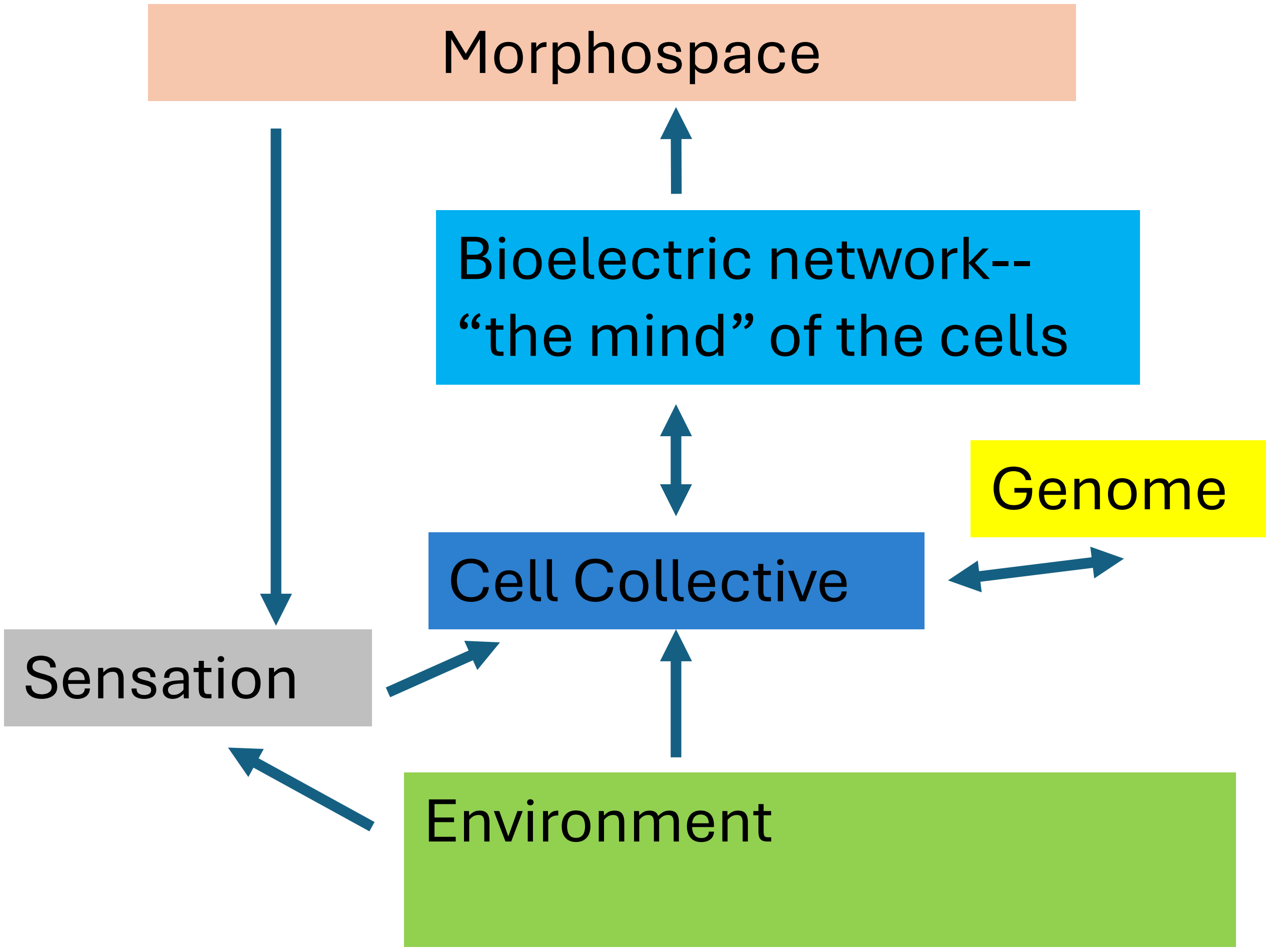
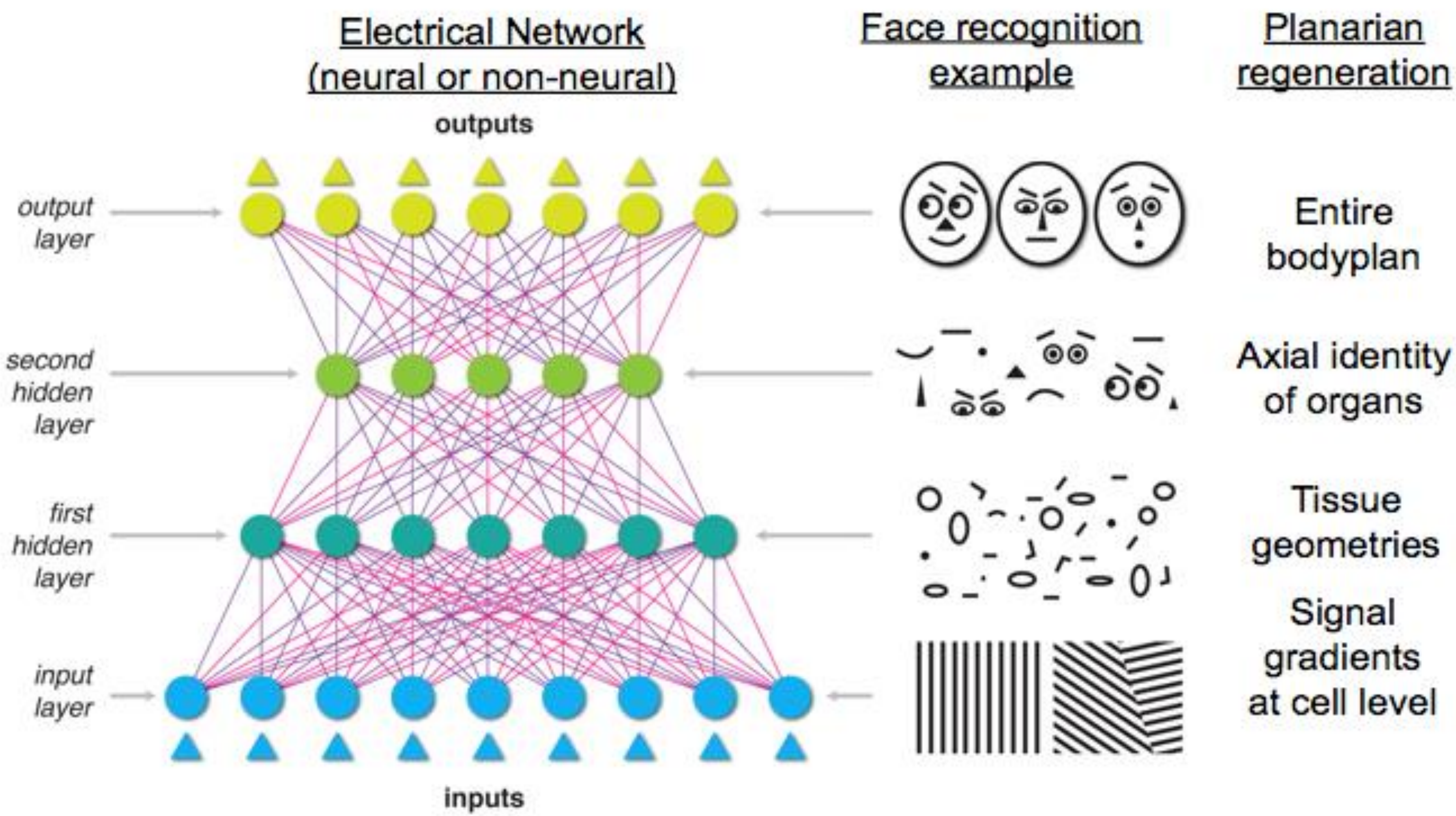
How could groups of cells do this? They don't have minds—or do they?

- Mathematical forms are internally represented in the bioelectric network.
- The network tells cell what to do to actualize this mathematical form.
- Feedback from the environment, sensation and the group of cells to the cells can "know" when they've achieved the goal of producing this form.
- Feedback allows the network to accurately represent (or track) the form over time.



How Higher Levels could be represented in bioelectrical networks



(Slide courtesy of Michael Levin; drawing by Jeremy Guay)

Primitive mathematical abilities

- This model of the relationship between cells and morphospace suggests the cells may have some primitive mathematical abilities because they need to be able to navigate through morphospace and to do this they need "concepts" of space and time.
- Mathematical abilities may be very basic to life and scale up to the human level of cognition.
- There is already a growing body of evidence for this; that other species, including primates, birds, bees, and even slime molds have mathematical abilities.
- Natural selection favors the ability to navigate through space and time.

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Office 365 stock image

https://www.sci.news/biology/slime-mold-problems-linear-time-06759.html

Conclusion

- During morphogenesis, multicellular organisms can use bioelectric networks to access, internally represent, and utilize information from a structured space of possible forms (“Platonic space”).
- This information, which can be understood as an Aristotelian formal cause of morphogenesis, structures morphogenesis by delineating a mathematical space of possible morphologies and by constraining the process of morphogenesis.
- The goals pursued by organisms during morphogenesis, which can be viewed as Aristotelian final causes can be characterized mathematically in terms of the morphospace itself or the electrochemical space of bioelectric network.
- This view is compatible with and compliments, more traditional views in which complex forms emerge from the bottom-up via the mathematics the underlies physics and chemistry.

Questions for further research

- How can morphospace be modelled mathematically? What are some different ways of doing this?
- How can electrochemical space be modelled mathematically? What are some different ways of doing this?
- What is the relationship—statistical, causal, or otherwise—between morphospace and electrochemical space? How can these two spaces be mapped onto each other?
- What is the relationship—statistical, causal, or otherwise—between electrochemical space and morphology? How can these two spaces be mapped onto each other? How, precisely, do bioelectric networks regulate cell behavior?
- How can we use our understanding of morphospace to make predictions about the electrochemical space and subsequent behavior multicellular organisms?

Questions for further research

- Do bioelectric networks have set points? How are these determined? How strong or weak or they? What affects this? Can setpoints be undone? How?
- How has the use of bioelectricity by cells evolved over time? What environmental and other pressures impact this process?
- Do microscopic organisms have mathematical abilities? Could a xenobot solve a maze or be trained to distinguish between numbers of objects?
- Can the idea of Platonic space be extended to lower and higher levels of organization, such as cells, genetic-regulatory networks, organ systems, animal behavior, and ecosystems? What is the relationship between these different Platonic spaces?
- What are some practical applications of this research program in synthetic biology, regenerative medicine, oncology, and biorobotics?

Acknowledgments

- Michael Levin
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