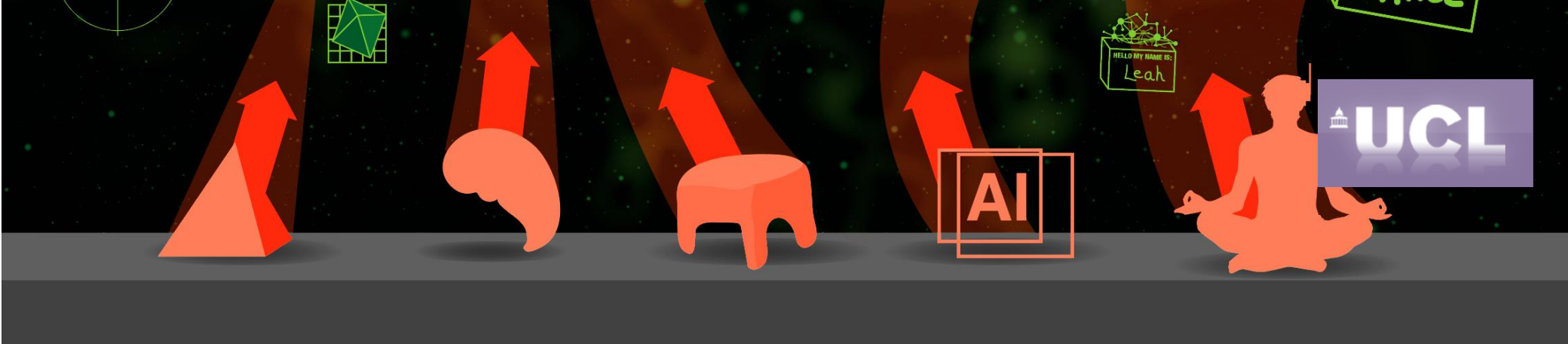




On the (Platonic) nature of things

Karl Friston

Abstract: This presentation offers an informal review of Platonic forms as seen through the lens of the free energy principle. It addresses the question "what is a state of being?" And answers this question through appeal to a recursive definition of the state of things at successive scales. Mathematically, this is expressed via the renormalization group. The implicit scale-invariance is taken as a foundational structure for the nature of things. I will unpack this perspective using a couple of instances in which it has been applied theoretically in the context of theoretical biology, empirically in the context of characterizing neuroimaging timeseries, and in machine learning, in the form of renormalizing generative models.

Key words: *active inference · autopoiesis · cognitive · dynamics · free energy · epistemic value · self-organization.*

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The statistics of life

Markov blankets, Bayesian mechanics
and simulations of a primordial soup

Renormalization group

From states to particles, and back again

Examples

Empirical, theoretical and practical

Coda

Goldilocks and dissipative structures

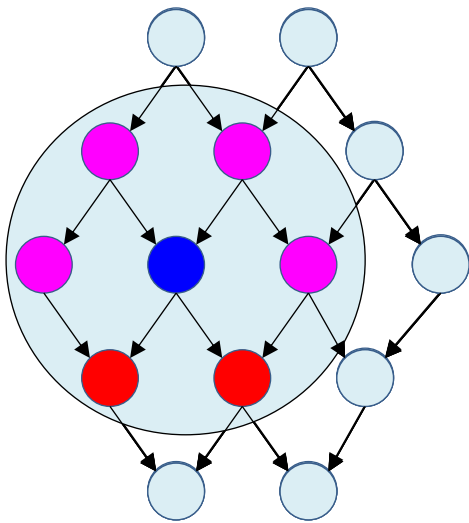


“How can the events in space and time which take place within the spatial boundary of a living organism be accounted for by physics and chemistry?”

(Erwin Schrödinger 1943)

The Markov blanket as a statistical boundary

(parents, children and parents of children)



η External states

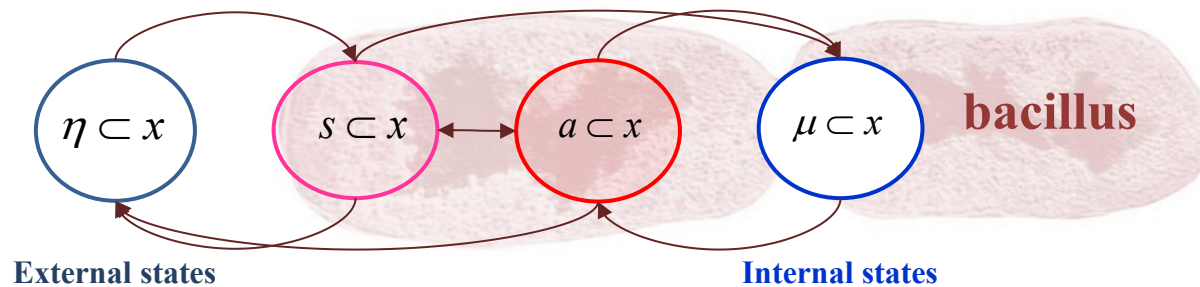
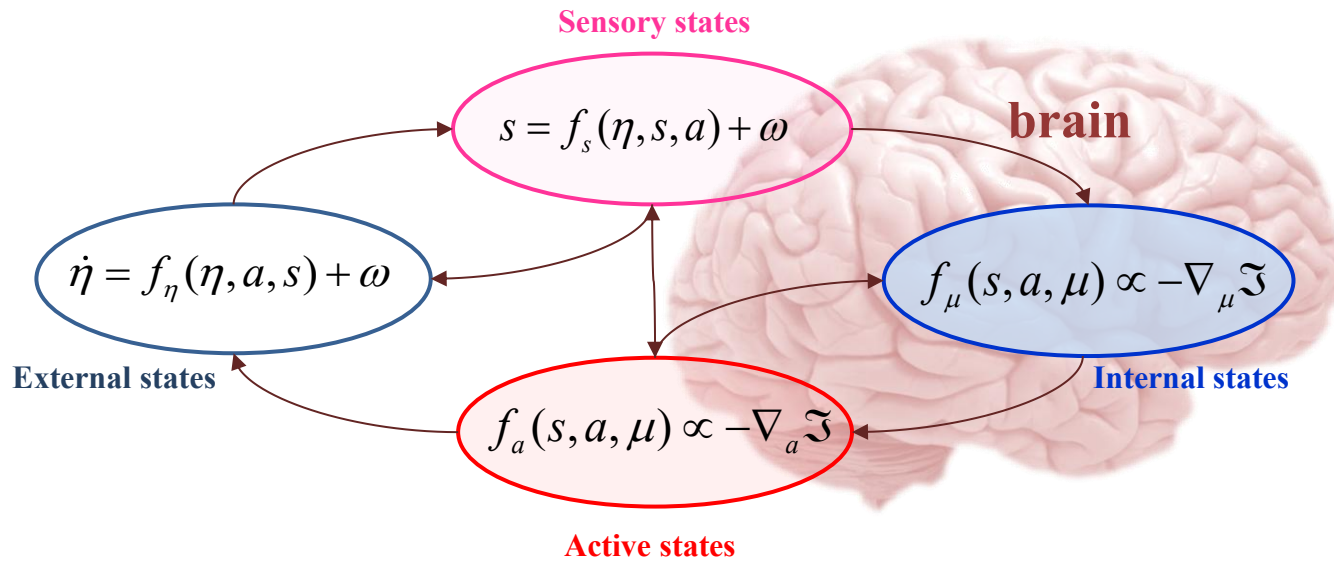
μ Internal states

s Sensory states

a Active states

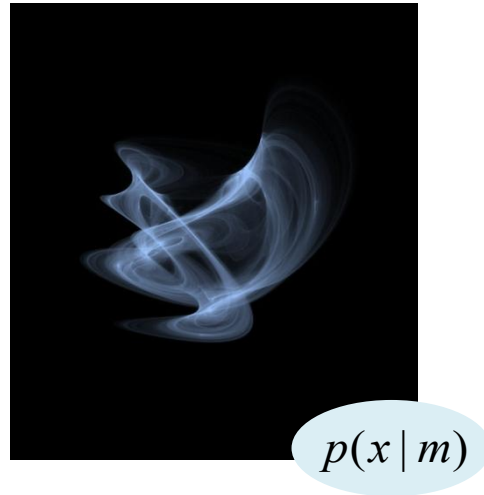
} Self-states (of a dissipative structure)

Markov blankets everywhere



Random dynamic systems and pullback attractors

$$\dot{x} = f(x) + \omega$$

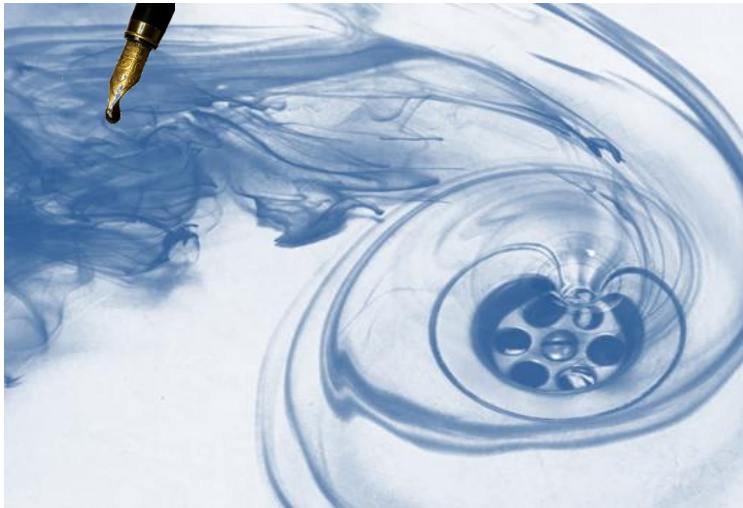


The Fokker-Planck equation $\dot{p}(x | m) = \nabla \cdot (\Gamma \nabla - f) p$

And its solution in terms of curl-free and divergence-free components

$$\dot{p}(x | m) = 0 \Leftrightarrow f(x) = (\Gamma - Q) \nabla \ln p(x | m)$$

The dynamics (i.e., flow) at NESS



Conservative

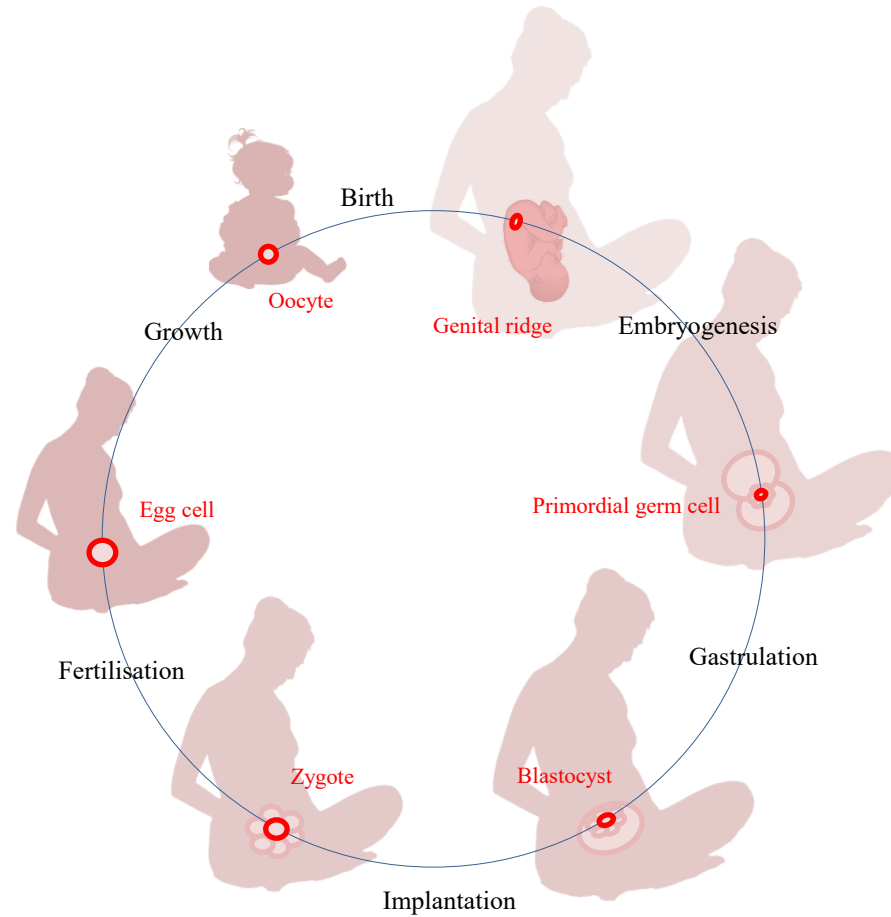
$$f_Q = -Q \cdot \nabla \ln p(x | m)$$

$$f_\Gamma = \Gamma \cdot \nabla \ln p(x | m)$$

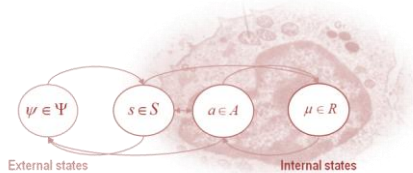
Dissipative

$$f = f_\Gamma + f_Q = (\Gamma - Q) \nabla \ln p(x | m)$$

Life cycles (from one point of view)



But what about the Markov blanket?



$$\dot{\boldsymbol{\mu}} = (\Gamma - Q) \nabla_{\boldsymbol{\mu}} \ln p(s | m)$$

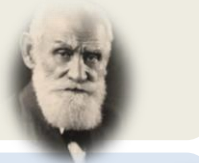
Perception

$$\dot{\mathbf{a}} = (\Gamma - Q) \nabla_{\mathbf{a}} \ln p(s | m)$$

Action

$$-F(s, \boldsymbol{\mu}) = \ln p(s | m) = \text{Value}$$

Reinforcement learning
Optimal control theory
Expected utility theory



$$F(s, \boldsymbol{\mu}) = -\ln p(s | m) = \text{Surprise}$$

Infomax principle
Minimum redundancy
Free-energy principle

Pavlov



$$\mathbb{E}[F(s, \boldsymbol{\mu})] = \mathbb{H}[p(s | m)] = \text{Entropy}$$

Self-organization
Synergetics
Homoeostasis

Barlow



$$p(s | m) = \text{Evidence}$$

Bayesian brain
Evidence accumulation
Predictive coding

Haken



Helmholtz

 Open Access

 Check for updates

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Research articles

Life as we know it

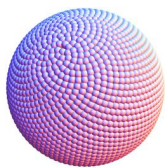
Karl Friston 

Published: 06 September 2013 | <https://doi.org/10.1098/rsif.2013.0475>

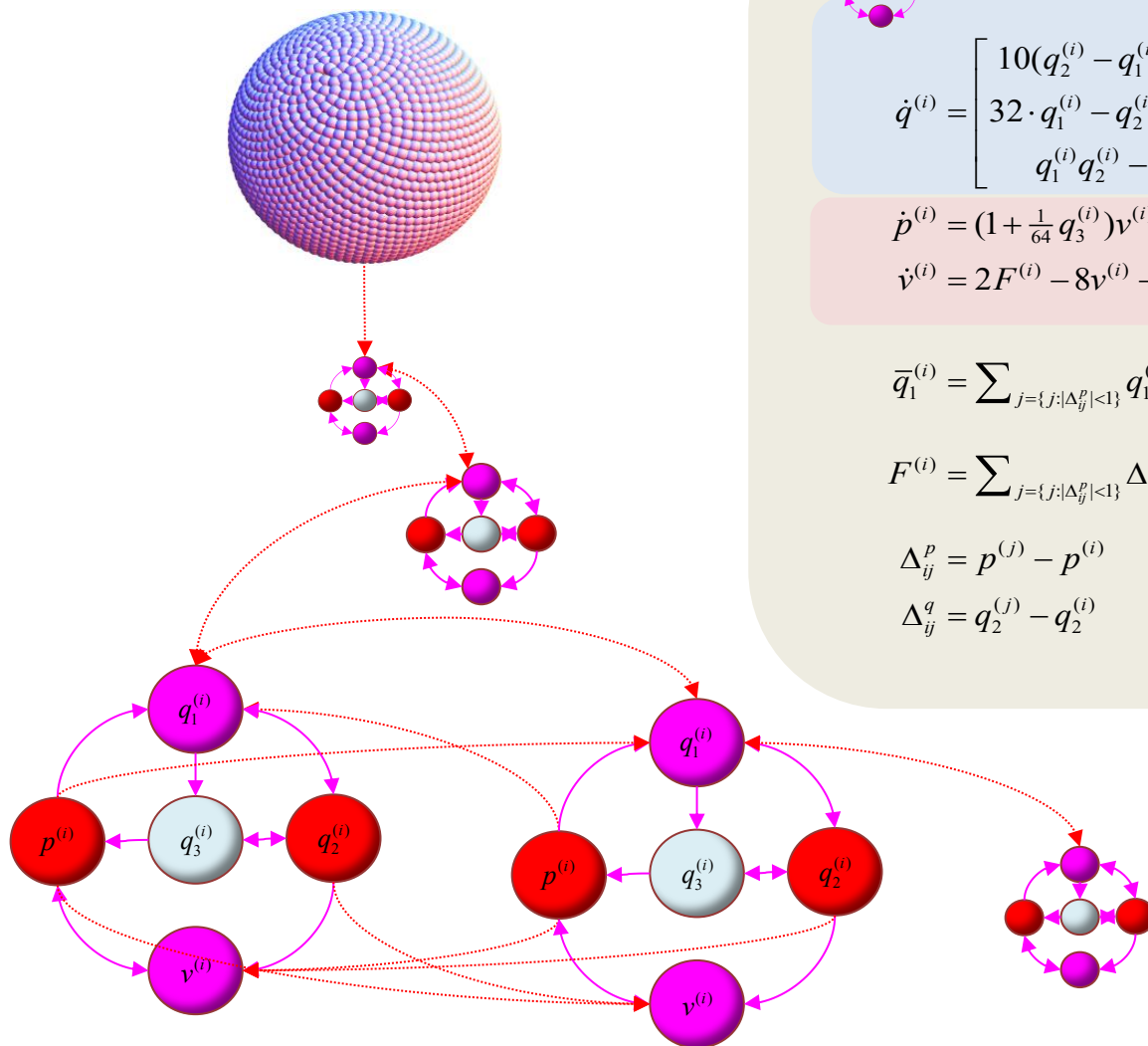


The statistics of life

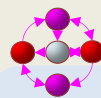
simulations of a primordial soup



Synthetic soup (active matter)



Equations of motion



Electrochemical

$$\dot{q}^{(i)} = \begin{bmatrix} 10(q_2^{(i)} - q_1^{(i)}) + \bar{q}_1^{(i)} \\ 32 \cdot q_1^{(i)} - q_2^{(i)} - q_3 q_1^{(i)} \\ q_1^{(i)} q_2^{(i)} - \frac{8}{3} q_3^{(i)} \end{bmatrix} \cdot \kappa^{(i)} + \omega_q$$

$$\dot{p}^{(i)} = (1 + \frac{1}{64} q_3^{(i)}) v^{(i)} + \omega_p$$

$$\dot{v}^{(i)} = 2F^{(i)} - 8v^{(i)} - p^{(i)} + \omega_v$$

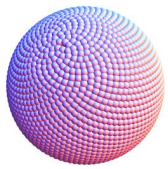
Newtonian

$$\bar{q}_1^{(i)} = \sum_{j=\{j: |\Delta_{ij}^p| < 1\}} q_1^{(j)}$$

$$F^{(i)} = \sum_{j=\{j: |\Delta_{ij}^p| < 1\}} \Delta_{ij}^p \left(\frac{8 \exp(-|\Delta_{ij}^q|) - 4}{|\Delta_{ij}^p|^2} - \frac{1}{|\Delta_{ij}^p|^3} \right)$$

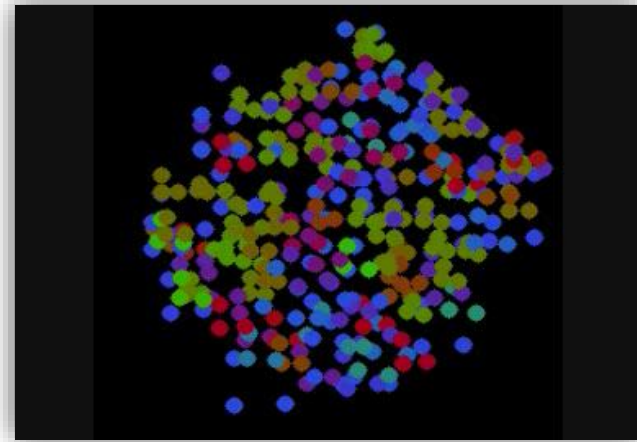
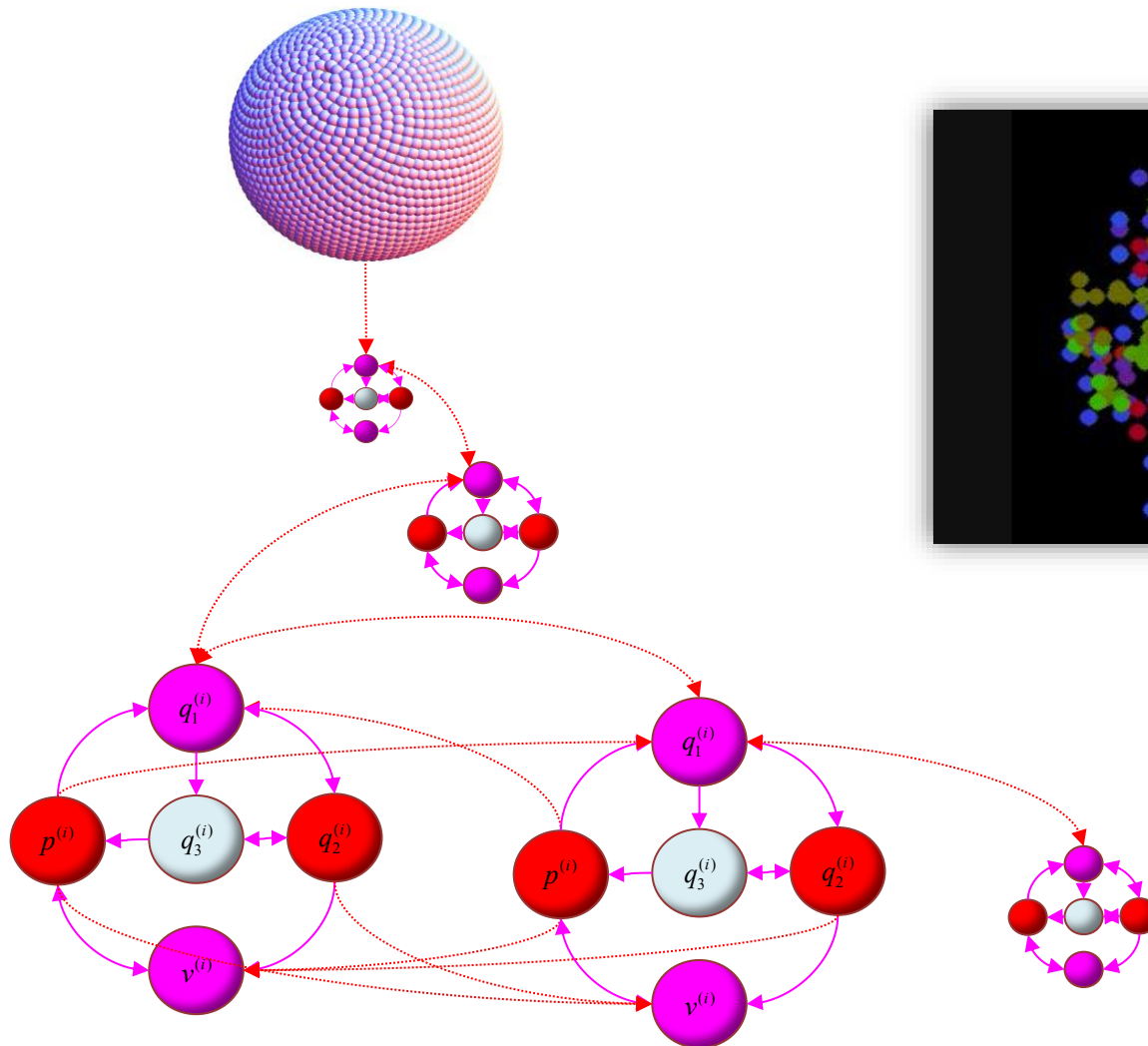
$$\Delta_{ij}^p = p^{(j)} - p^{(i)}$$

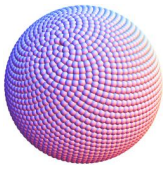
$$\Delta_{ij}^q = q_2^{(j)} - q_2^{(i)}$$



Synthetic soup (active matter)

Simulations of a (prebiotic) primordial soup





Finding the Markov blanket

The Jacobian encodes the children, parents and parents of children

$$B = J + J^T + J^T J$$

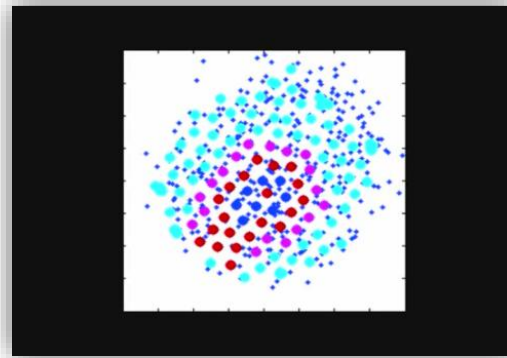
$$\text{Markov Blanket} = B \cdot [\text{eig}(B) > u]$$

Hidden states

Sensory states

Active states

Internal states



Is this a particle?

**A free energy principle for a particular physics**

Karl Friston



The renormalization group, from states to particles, and back again

Now, let us ask a more fundamental question: what is a state? This question can be dissolved by appealing to an infinite regress along the following lines:

- *What is a state? A state is an eigenstate of a particle's Markov blanket.*
- *What is a particle? A particle is a set of blanket and internal states.*
- *What is a state? A state is ... and so on.*

An eigenstate here refers to the amplitude of an eigenmode of blanket states; namely, the principal eigenvectors of their Jacobian (i.e., rate of change of flow with respect to state). These mixtures are formally identical to *order parameters* in synergetics that reflect the amplitude of slow, unstable eigenmodes (Haken, 1983). In terms of centre manifold theory, they correspond to solutions on the slow (unstable or centre) manifold (Carr, 1981; Davis, 2006).

Scale-invariance and RG operators

Bayesian mechanics: the dynamics of anything can be described as active inference

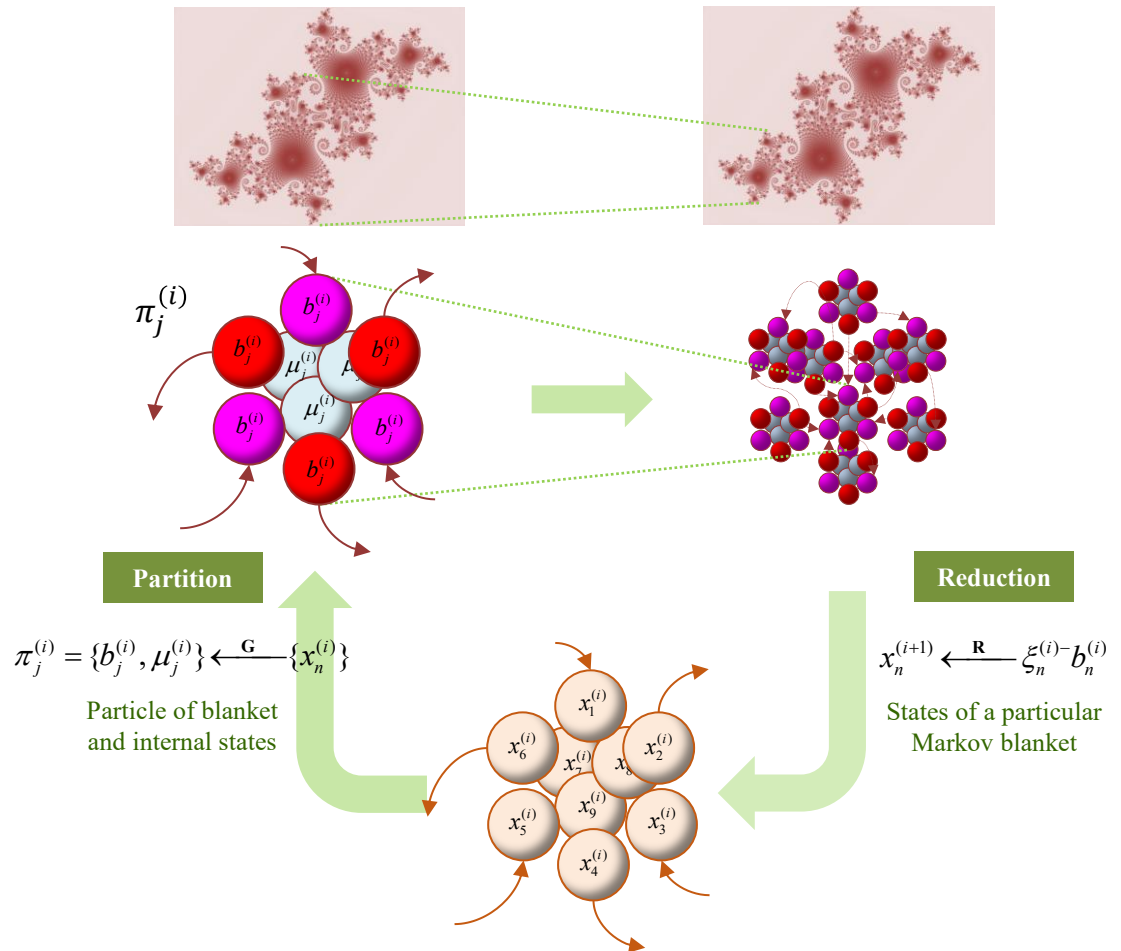
But before Bayesian mechanics....

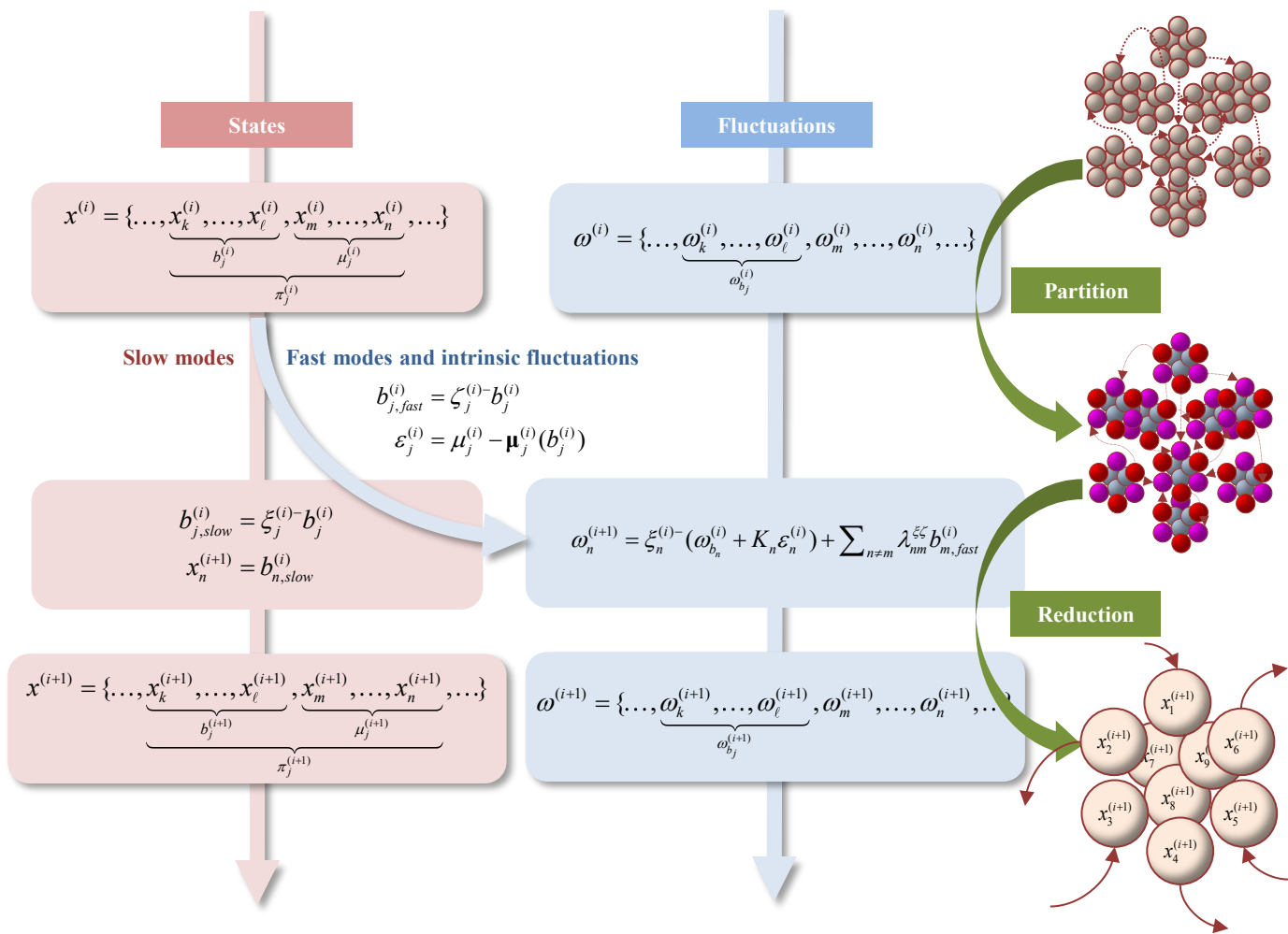
- Particles: a set of (microscopic) blanket and internal states
- States: a set of (macroscopic) Eigen (self) functions of blanket states

$$\dot{x}_n^{(i)} = f_n^{(i)} + \sum_m \lambda_{nm}^{(i)} x_m^{(i)} + \omega_n^{(i)}$$

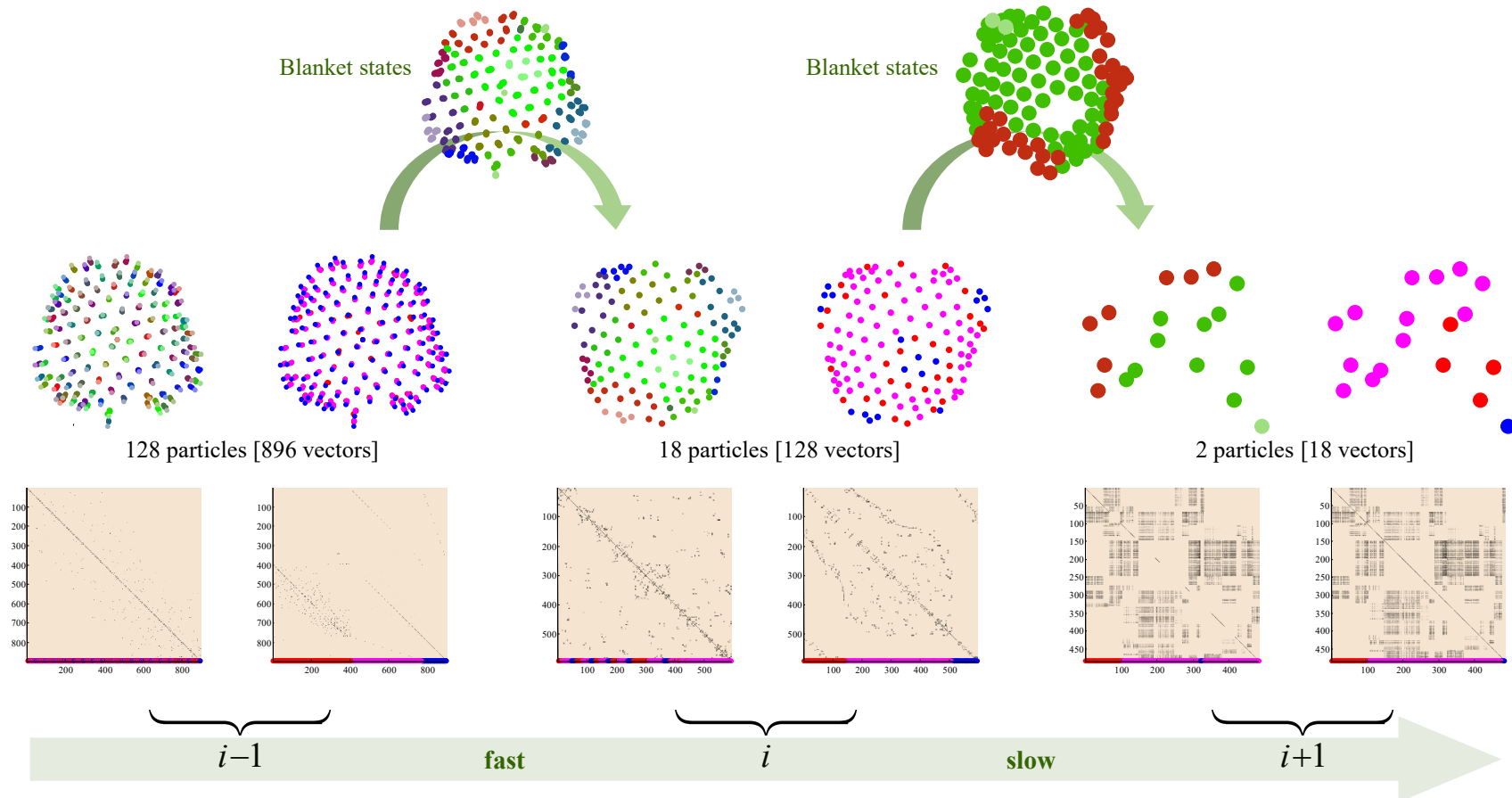
$$x^{(i)} = \{x_1^{(i)}, \dots, x_N^{(i)}\}$$

$$\mathbb{E}[\omega_n^{(i)}(\tau) \cdot \omega_m^{(i)}(\tau')] = \begin{cases} 2\Gamma_n^{(i)} \delta(\tau - \tau') & n = m \\ 0 & n \neq m \end{cases}$$





Hierarchical decomposition





March 01 2021

Parcels and particles: Markov blankets in the brain

In Special Collection: CogNet

Karl J. Friston, Erik D. Fagerholm, Tahereh S. Zarghami, Thomas Parr, Inés Hipólito, Loïc Magrou, Adeel Razi

Check for updates

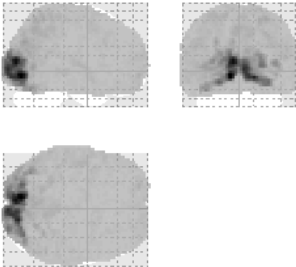
Author and Article Information

Network Neuroscience (2021) 5 (1): 211–251.

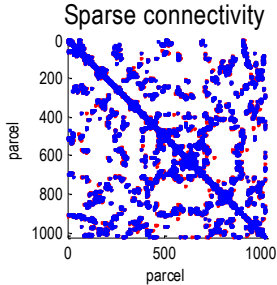
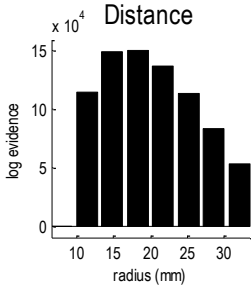
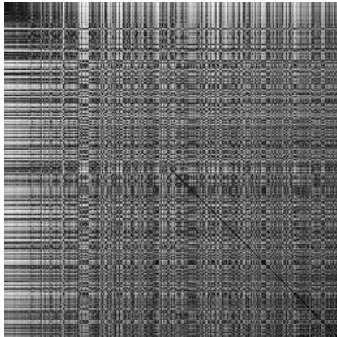
https://doi.org/10.1162/netn_a_00175 Article history

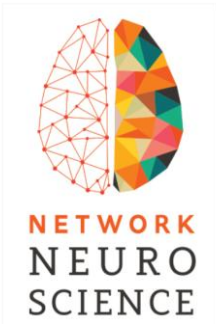


Variance of voxels

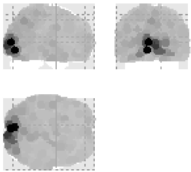


Distance between particles

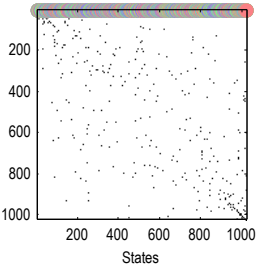




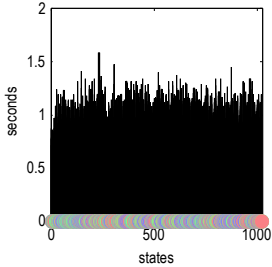
1024 states, 1024 particles
(1024 eigenstates)



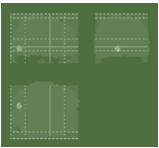
Jacobian



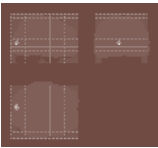
Time constants



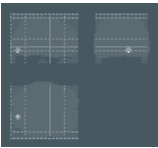
Particle 1 of 1024 (0,0,1)
1 eigenmodes



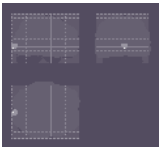
Particle 2 of 1024 (0,0,1)
1 eigenmodes



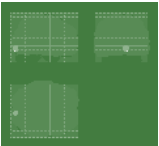
Particle 3 of 1024 (0,0,1)
1 eigenmodes



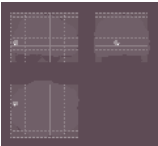
Particle 4 of 1024 (0,0,1)
1 eigenmodes



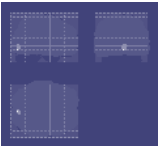
Particle 5 of 1024 (0,0,1)
1 eigenmodes



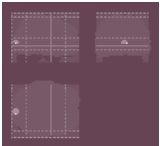
Particle 6 of 1024 (0,0,1)
1 eigenmodes



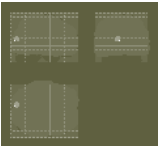
Particle 7 of 1024 (0,0,1)
1 eigenmodes



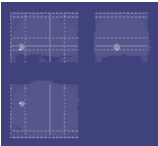
Particle 8 of 1024 (0,0,1)
1 eigenmodes



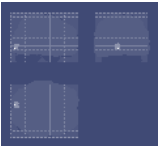
Particle 9 of 1024 (0,0,1)
1 eigenmodes



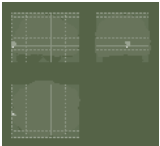
Particle 10 of 1024 (0,0,1)
1 eigenmodes

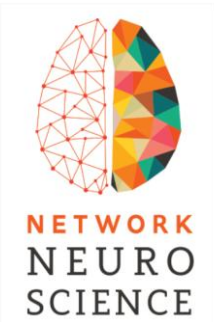


Particle 11 of 1024 (0,0,1)
1 eigenmodes

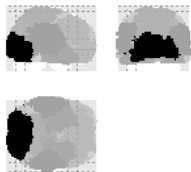


Particle 12 of 1024 (0,0,1)
1 eigenmodes

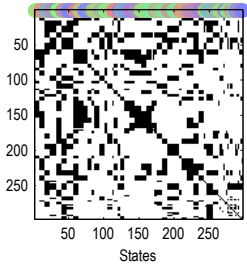




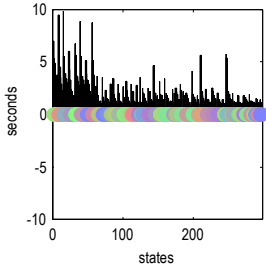
1024 states, 57 particles
(296 eigenstates)



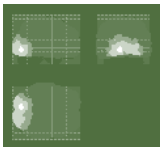
Jacobian



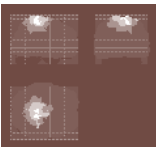
Time constants



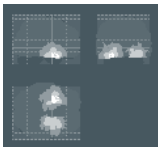
Particle 1 of 57 (1,29,44)
8 eigenmodes



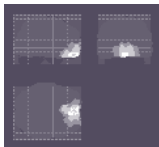
Particle 2 of 57 (1,13,36)
7 eigenmodes



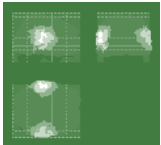
Particle 3 of 57 (1,15,49)
8 eigenmodes



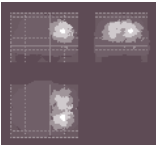
Particle 4 of 57 (1,11,20)
8 eigenmodes



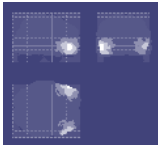
Particle 5 of 57 (1,16,65)
8 eigenmodes



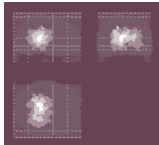
Particle 6 of 57 (1,26,58)
8 eigenmodes



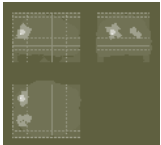
Particle 7 of 57 (1,8,28)
8 eigenmodes



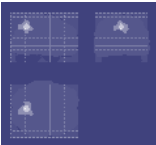
Particle 8 of 57 (1,12,58)
8 eigenmodes



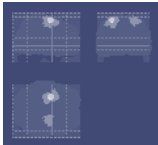
Particle 9 of 57 (0,1,13)
7 eigenmodes



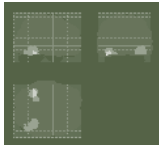
Particle 10 of 57 (0,1,9)
5 eigenmodes

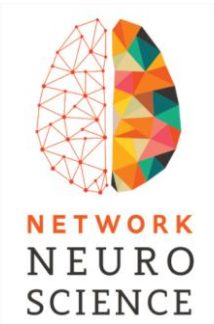


Particle 11 of 57 (0,1,11)
7 eigenmodes

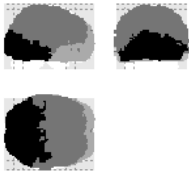


Particle 12 of 57 (0,1,9)
4 eigenmodes

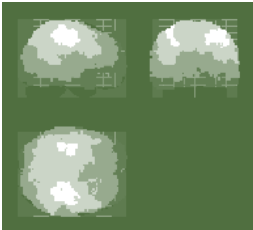
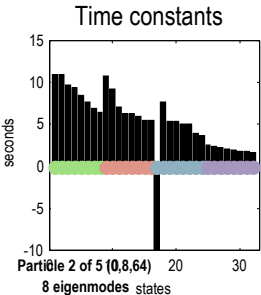
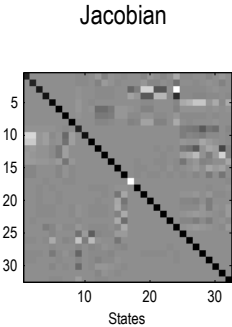




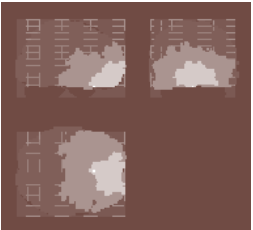
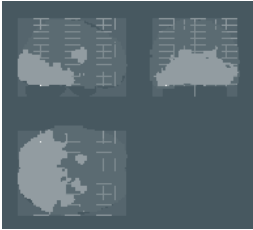
296 states, 5 particles
(32 eigenstates)



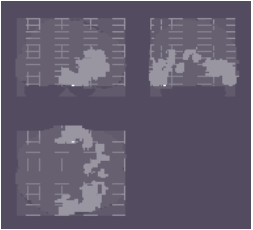
Particle 1 of 5 (6,71,87)
8 eigenmodes



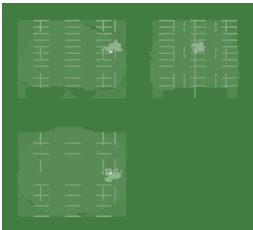
Particle 3 of 5 (0,0,38)
8 eigenmodes

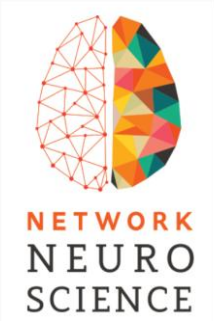


Particle 4 of 5 (0,0,21)
8 eigenmodes



Particle 5 of 5 (0,0,1)
0 eigenmodes

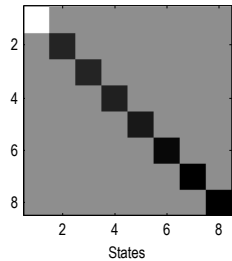




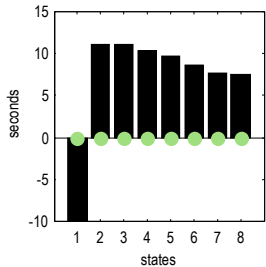
32 states, 1 particles
(8 eigenstates)



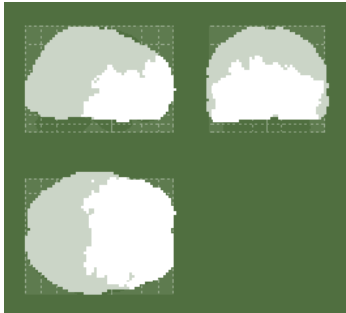
Jacobian



Time constants

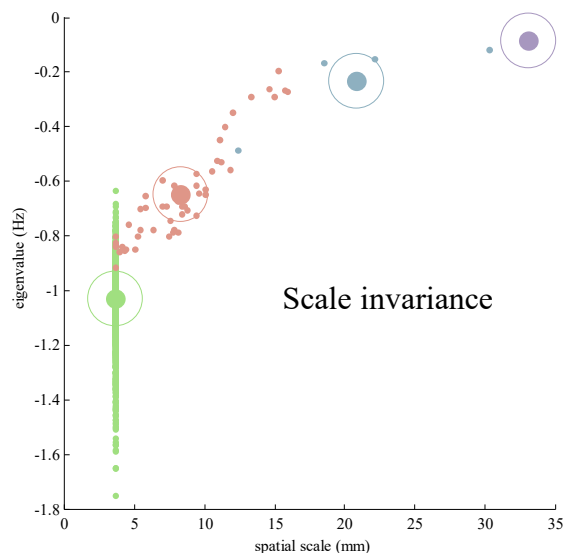


Particle 1 of 1 (8,24,0)
8 eigenmodes





Scale-invariance and RG flows

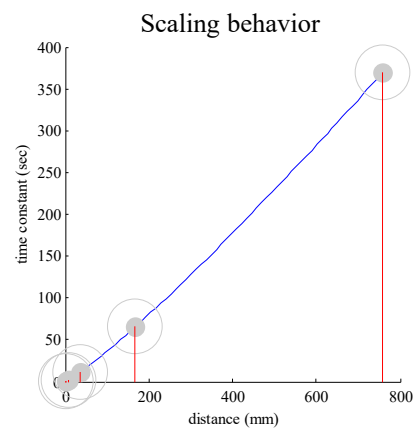
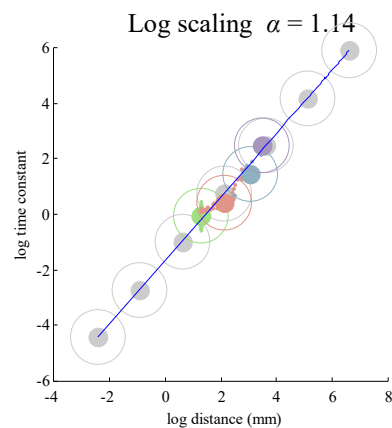


Consider the following RG flow or beta function:

$$\sigma_{\tau}^{(i+1)} = e^{\beta_{\tau}} \sigma_{\tau}^{(i)}$$

$$\sigma_{\tau}^{(i)} = E[-\text{Re}(\lambda_{nn_j}^{(i)})]$$

This says as we move from one scale to the next, the time scale increases by $e^{\beta_{\tau}} \geq 1$.

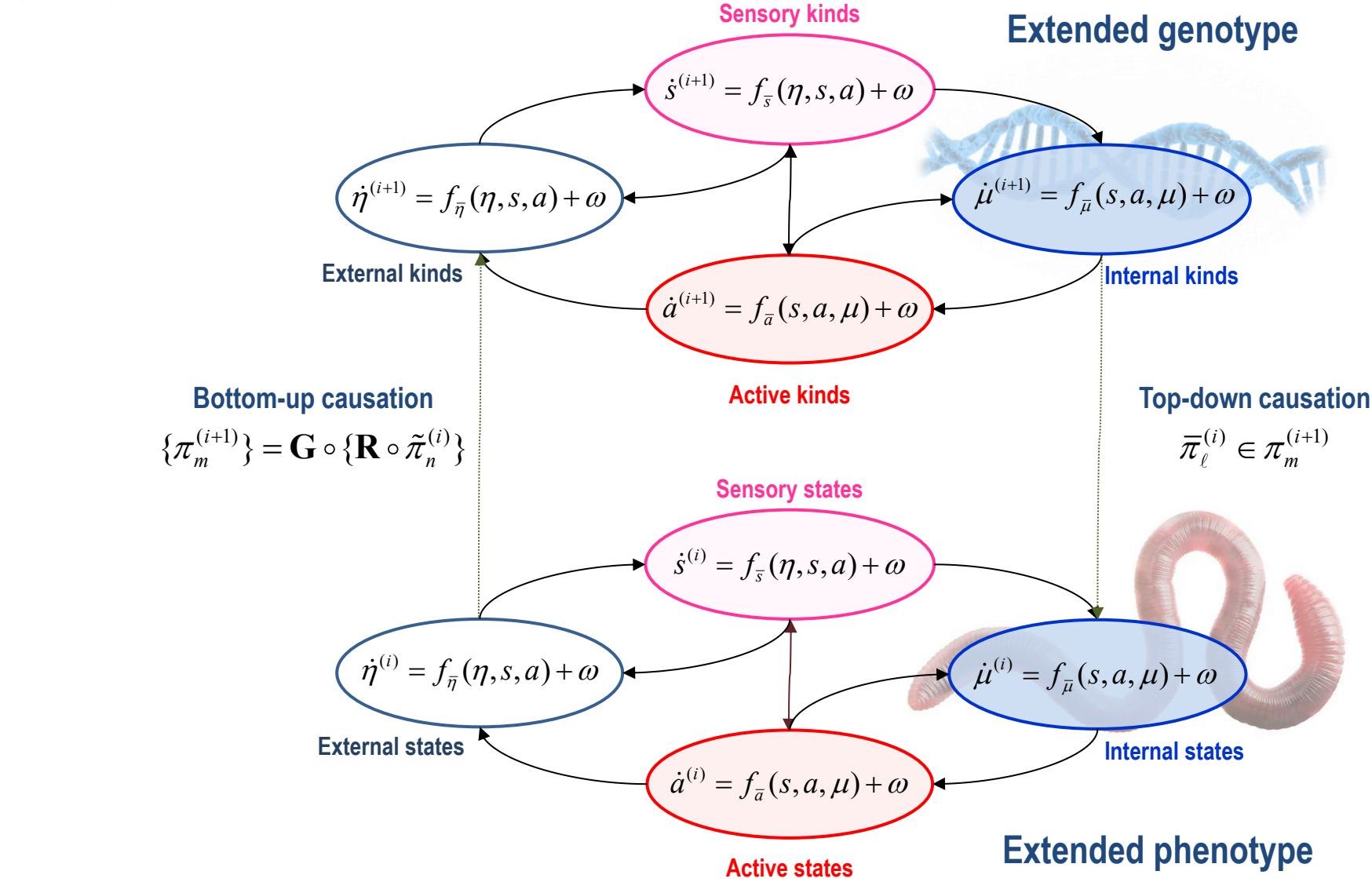




NETWORK
NEURO
SCIENCE

Generative models and spatiotemporal scales

Scale	Spatial scale	Time scale	Example
-8	4.38 μm	380 μs	Dendritic spines occur at a density of up to 5 spines per μm of dendrite. Spines contain fast voltage-gated ion channels with time constants in the order of 1 ms.
-4	89.3 μm	11.9 ms	A cortical minicolumn: a minicolumn measures of the order of 40–50 μm in transverse diameter 80 μm spacing (Peters & Yilmaz, 1993). The membrane time constant of a typical cat layer III pyramidal cell is about 20 ms.
0	1.82 mm	374 ms	A cortical hypercolumn (e.g., a 1 mm expanse of V1 containing ocular dominance and orientation columns for a particular region in visual space (Mountcastle, 1997)). Typical duration of evoked responses in the order of 1 to 300 ms (c.f., the cognitive moment).
4	37.2 mm	11.8 sec	The cerebellum is about 50 mm in diameter, corresponding to the size of cortical lobes. Sympathetic unit activity associated with Mayer waves within frequency of 0.1 Hz (wavelength of 10 seconds).
8	758 mm	6.15 min	A dyadic interaction (e.g., a visit to your doctor).
12	15.5 m	3.22 hrs	A dinner party for six guests, lasting for several hours.
16	.31 km	4.21 days	An international scientific conference (pre-coronavirus).



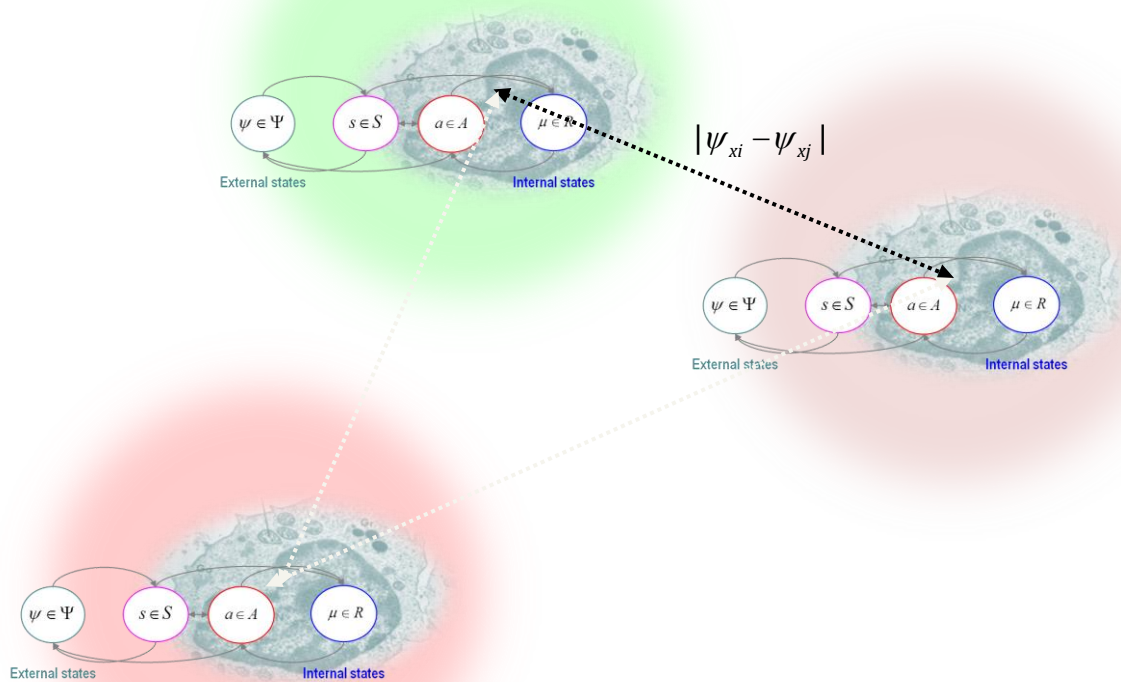
Knowing one's place: a free-energy approach to pattern regulation

Karl Friston¹, Michael Levin², Biswa Sengupta¹ and Giovanni Pezzulo³

¹The Wellcome Trust Centre for Neuroimaging, Institute of Neurology, Queen Square, London, UK²Biology Department, Center for Regenerative and Developmental Biology, Tufts University, Medford, USA

³Institute of Cognitive Sciences and Technologies, National Research Council, Rome, Italy

$$\lambda_i(\psi_x, \psi_c) = \tau \prod_j \psi_{c_j} \exp(|\psi_{xi} - \psi_{xj}|)$$



Intercellular signaling

$$S = \begin{bmatrix} s_c \\ s_x \\ s_\lambda \end{bmatrix} = \begin{bmatrix} \psi_c \\ \psi_x \\ \lambda(\psi_x, \psi_c) \end{bmatrix} + \omega$$

Intrinsic signals

Exogenous signals

Endogenous signals

Generative model

$$g(\psi) = \begin{bmatrix} \psi_c^* \\ \psi_x^* \\ \lambda(\psi_x^*, \psi_c^*) \end{bmatrix} \sigma(\psi)$$

$$\sigma(\psi_i) = \frac{\exp(\psi_i)}{\sum_j \exp(\psi_j)}$$

Knowing one’s place: a free-energy approach to pattern regulation

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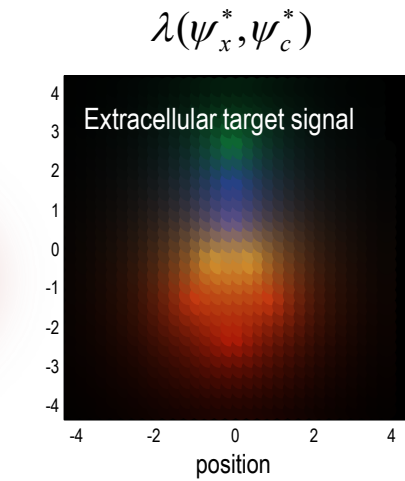
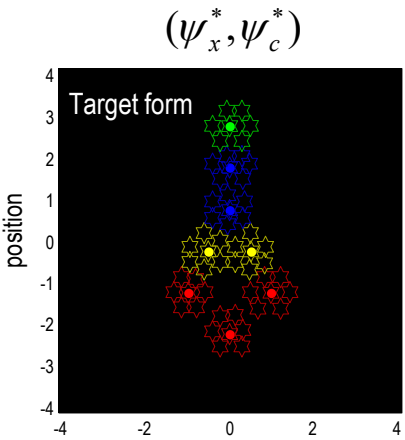
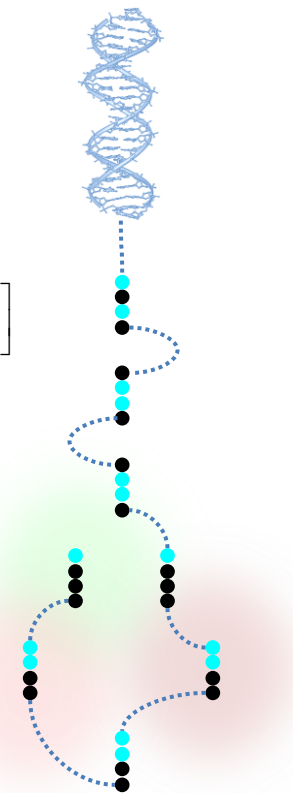
(Genetic) encoding of target morphology

Position (place code)

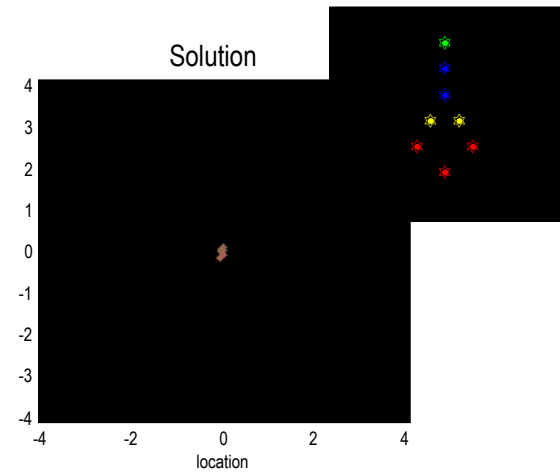
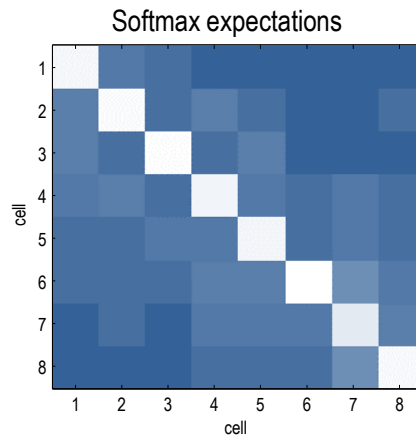
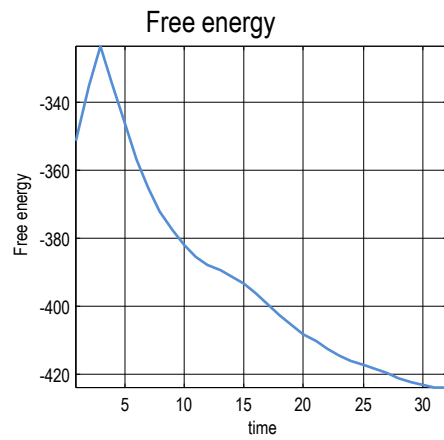
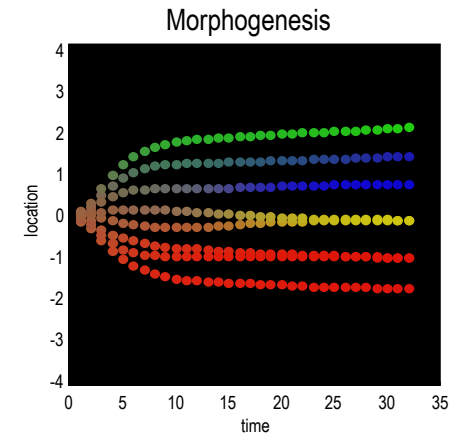
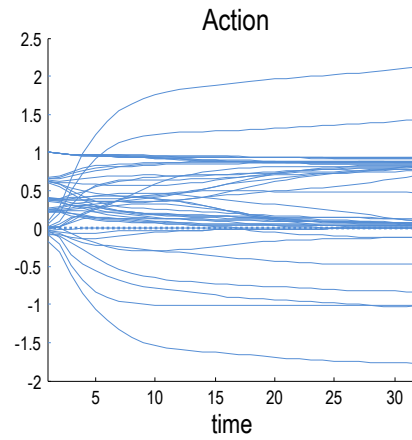
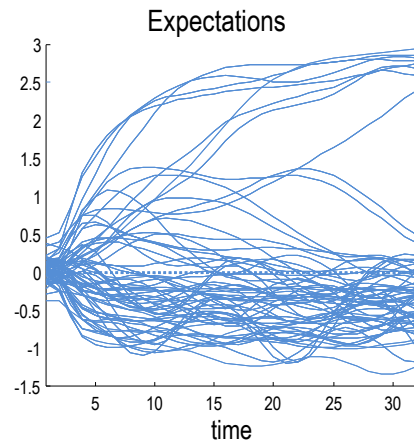
$$\psi_x^* = \frac{1}{4} \begin{bmatrix} -9 & -5 & -5 & -1 & -1 & 3 & 7 & 11 \\ 0 & -4 & 4 & -2 & 2 & 0 & 0 & 0 \end{bmatrix}$$

Signal expression (genetic code)

$$\psi_c^* = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$



Morphogenesis





From pixels to planning: scale-free active inference

Karl Friston^{1,2*}, Conor Heins², Tim Verbelen²,
Lancelot Da Costa^{1,2}, Tommaso Salvatori², Dimitrije Markovic^{2,3},
Alexander Tschantz², Magnus Koudahl², Christopher Buckley^{2,4}
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¹Queen Square Institute of Neurology, University College London, London, United Kingdom, ²VERSES
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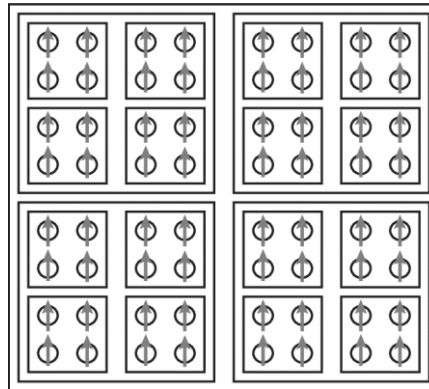


Corpus ID: 215316

An exact mapping between the Variational Renormalization Group and Deep Learning

Pankaj Mehta, D. Schwab • Published in [arXiv.org](https://arxiv.org) 14 October 2014 • Computer Science, Physics

TLDR This work constructs an exact mapping from the variational renormalization group, first introduced by Kadanoff, and deep learning architectures based on Restricted Boltzmann Machines (RBMs), and suggests that deep learning algorithms may be employing a generalized RG-like scheme to learn relevant features from data. [Expand](#)



Parametric depth

$$P(o_\tau | s_\tau, a) = \text{Cat}(\mathbf{A})$$

$$P(s_{\tau+1} | s_\tau, u_\tau, b) = \text{Cat}(\mathbf{B})$$

$$P(u_{\tau+1} | u_\tau, c) = \text{Cat}(\mathbf{C})$$

$$P(s_0 | d) = \text{Cat}(\mathbf{D})$$

$$P(u_0 | e) = \text{Cat}(\mathbf{E})$$

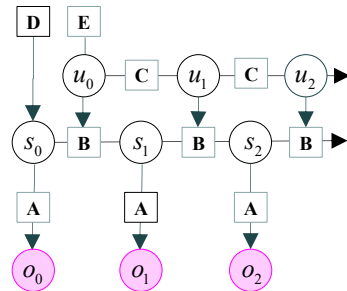
$$P(\mathbf{A}) = \text{Dir}(a)$$

$$\mathbb{E}[P(\mathbf{A})] =: \mu(a) = a/a_0 : a_0 = \sum_i a_i$$

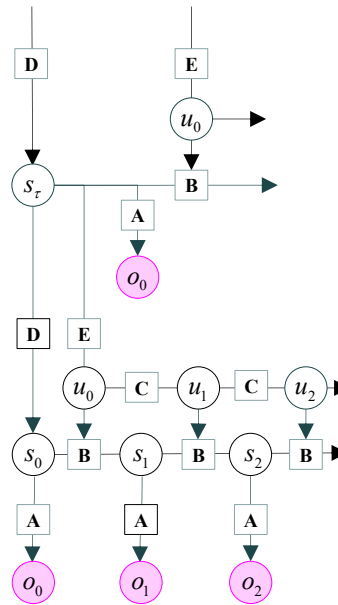
$$\mathbb{E}[\ln P(\mathbf{A})] =: \varphi(a) = \psi(a) - \psi(a_0)$$

$$\ell(a) =: \ln(a)$$

$$\sigma(a) =: \mu(e^a)$$

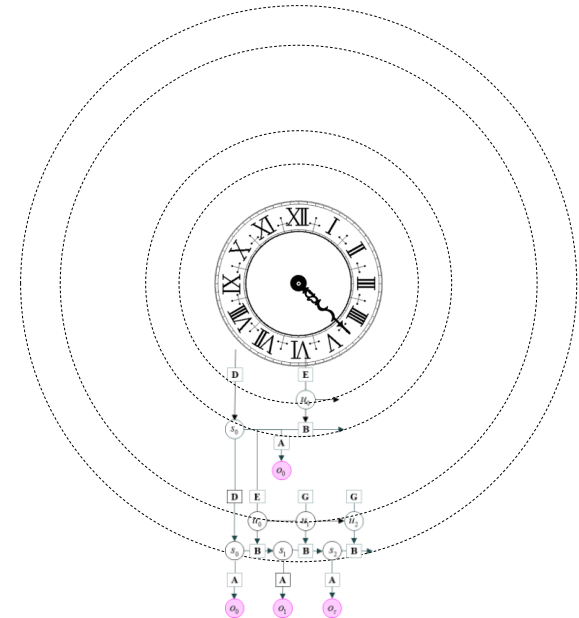


Temporal depth

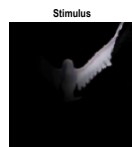
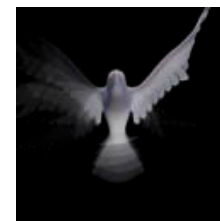
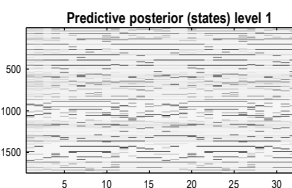
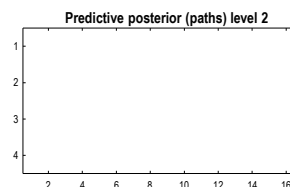
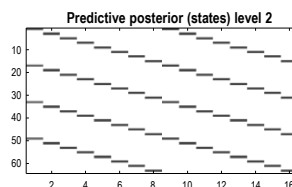
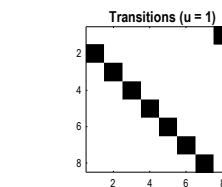
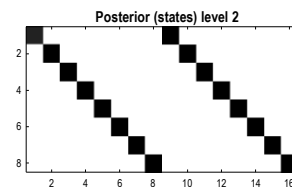
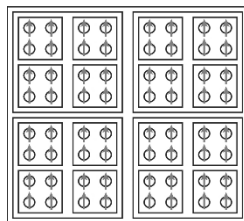
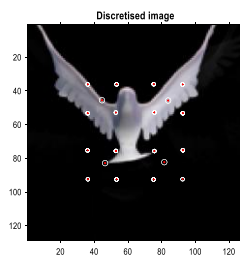
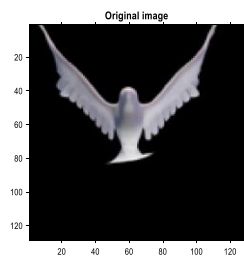


Hierarchical depth

Generalised discrete state
space models
(with temporal dilation)



Generative AI and image completion





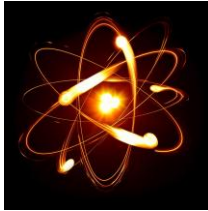
Coda

Turtles all the way down?



Self-organisation and mechanics

$$\Gamma \gg Q$$

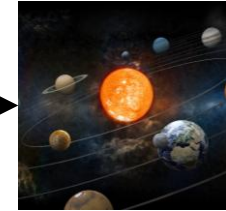


$$f = (\Gamma - Q) \nabla \ln p(x')$$

scale



$$\Gamma \ll Q$$



Complex (Quantum) systems

$$Q \rightarrow 0$$

$$p = \Psi \cdot \Psi^\dagger$$

$$\Psi = \exp(-\frac{1}{2\Gamma} U - \frac{i}{\hbar} \mathbf{E} t)$$

$$f(x) = -\nabla U \Rightarrow U = -\Gamma \ln p$$

$$\Gamma = \frac{\hbar^2}{2m}$$

$$\varphi(x) = \frac{1}{2} (\frac{1}{2\Gamma} \nabla U \cdot \nabla U - \nabla^2 U)$$

$$-\frac{\hbar^2}{2m} \cdot \nabla^2 \Psi + \varphi(x) \Psi = \mathbf{E} \Psi$$

Time-independent Schrödinger equation and quantum mechanics

Dissipative systems
(with detailed balance)

$$Q \rightarrow 0$$

$$f(x) = -\nabla U = \Gamma \nabla \ln p$$

$$\Gamma = k_B T$$

$$p(x | m) = \exp(-U/k_B T)$$

Integral fluctuation theorems
and statistical mechanics

Open systems
(with a Markov blanket)

$$\dot{\boldsymbol{\mu}} = (Q - \Gamma) \nabla_{\boldsymbol{\mu}} F$$

$$\dot{\mathbf{a}} = (Q - \Gamma) \nabla_{\mathbf{a}} F$$

$$F(b, \boldsymbol{\mu}) = -\ln p(b, \boldsymbol{\mu})$$

$$\boldsymbol{\mu} = \mathbb{E}[\boldsymbol{\mu} | b]$$

$$q_{\boldsymbol{\mu}}(\eta) = p(\eta | b)$$

$$F(b, \boldsymbol{\mu}) \triangleq F[b, q_{\boldsymbol{\mu}}(\eta)]$$

Self-evidencing and Bayesian
mechanics

Conservative systems

$$\Gamma \rightarrow 0$$

$$\ln p(x, x' | m) = \frac{\varphi(x)}{m} + \frac{1}{2} x'^2$$

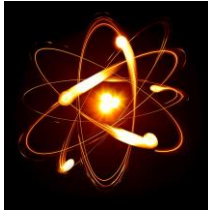
$$Q = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$f = -Q \nabla \ln p = \begin{bmatrix} x' \\ -\frac{1}{m} \nabla \varphi \end{bmatrix}$$

Newton's laws of motion and
classical (Lagrangian) mechanics

Self-organisation and mechanics

$$\Gamma \gg Q$$

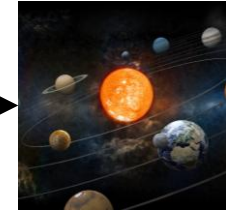


$$f = (\Gamma - Q) \nabla \ln p(x)$$

scale



$$\Gamma \ll Q$$



“Only when a system behaves in a sufficiently random way may the difference between past and future, and therefore irreversibility, enter its description.”

— Ilya Prigogine, *Order Out of Chaos: Man's New Dialogue with Nature*

Open systems
(with a Markov blanket)

$$\dot{\mu} = (Q - \Gamma) \nabla_{\mu} F$$

$$\dot{a} = (Q - \Gamma) \nabla_a F$$

$$F(b, \mu) = -\ln p(b, \mu)$$

$$\mu = \mathbb{E}[\mu | b]$$

$$q_{\mu}(\eta) = p(\eta | b)$$

$$F(b, \mu) \triangleq F[b, q_{\mu}(\eta)]$$

Self-evidencing and Bayesian
mechanics

Summary

The “*Siphonaptera*”, sometimes referred to as Fleas:

*Greater fleas have little fleas,
Upon their backs to bite 'em,
And little fleas have lesser fleas,
and so, ad infinitum.*



Complex systems do not forget their initial conditions: they “carry their history on their backs”

— *Prigogine, Spring 1995, US Naval Academy.*

Thank you

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